The IEAR Process in image coding

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
Master of Technology

by

ASHUTOSH DIXIT

to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

April, 1990.

CERTIFICATE

It is certified that the work contained in the thesis entitled ***The IEAR process in image coding*** by *Ashutosh Dixit*, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

S. K. Mullick
Professor
Department of Electrical Engineering
I.I.T., Kanpur.

April, 1990.

EE-1990-M-DIX-IEA

ACKNOWLEDGEMENTS

I wish to acknowledge the contributions of the following people towards the completion of this work:

Firstly, my deep-felt sense of gratitude towards the care and enthusiasm showed by Dr. Mullick and his guidance which shaped the growth of this work.

I also wish to express my thanks to the following people who all helped, in their way: Mr. D.V.S.S.N. Murty for providing various facilities that made the computational work and prepartion of the thesis easier, Hariprasad and George in the IP lab and Pramod for helping with the snaps.

If I were allowed to pluck out a few names, out of the intermittent-sense-of-fraternity-that-enveloped-me, I would choose BK, Sujoy, Fish, Mathur, Arun, DD and Sudhir.

And last, someone in me who barricaded the reserves of my patience, scruples and nicotine against the onslaughts of monotony.

ABSTRACT

Modern communication practices make extensive uses of image data. The forbidding volumes of such data make it mandatory that methods for image data compression be considered.

Certain models for describing non-gaussian time series have been studied in literature. Gibson, [1], considered the IEAR(1) process, a first order AR process in which the excitation is absent with a prabability p. It was shown that the structure of this process makes it amenable to data compression purposes, and a predictive variable length compression scheme for the coding of this process was designed. Here, a new process, IEAR-IC(1) process has been constructed, and the feasibility of these two processes in image source modelling and coding have been explored. A large number of configurations in which compression schemes based on these processes can exist in an image coding environment have been identified and the compression performances of various schemes compared. Both adaptive and non-adaptive methods have been considered. Comparison has been made with 3-bit DPCM.

It has finally been shown, using experimental evidence, that the scheme yielding the best overall compression performance turns out to be a non-adaptive scheme which codes the prediction error values, obtained on using a predictor coeffecient of value 1.0, using a simple variable length coding scheme. The above scheme turns out to be identical to the use of the same coding scheme in the case of a closely related DPCM scheme.

CONTENTS

Chapter 1

Introduction

Data compression is the conversion of a stream of analog or very high rate discrete data into a stream of relatively low rate data for communication over a digital communication link or storage into a digital memory. The need for parsimonious representations of data is exemplified by image data, where broadcast television with a resolution of 512x512 pels/frame requires a rate of nearly 60×10^6 bit/s at 30 frames/s and 8 bit/pel intensity resolution. These large memory and/or channel bandwidth requirements make it mandatory to consider data compression methods.

Such conversion of a relatively high rate data to lower rate data virtually always entails a loss of fidelity or an increase in distortion. The efficiency of a data compression algorithm is measured by its data compressing ability ,the resulting distortion as well as by its implmentation complexity. Hence the goal of a source coding system is to obtain the best possible fidelity for a given rate, or equivalently to minimize the rate for a given fidelity. The complexity of data compression algorithms is an important consideration in their hardware implementation.

Image data compression methods find their use in a variety of environments.

Applications in information transmission include broadcast television, remote sensing via satellite, radar, sonar, teleconferencing, computer communications, facisimile transmission, remotely piloted vehicles; and in information storage include educational and business documents, medical images used in patient monitoring systems and radioastronomy.

1.1 Review of Image coding methods

This section reviews, extremely briefly, the methodologies prevelant in source coding of images.

Image coding methods can be broadly classified into two subdivisions:

1) Point processing image coding

2) Spatial processing image coding

1) Point processing image coding

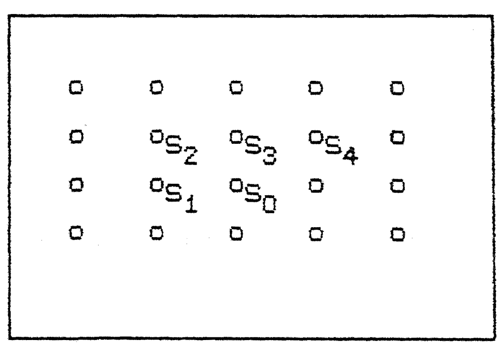
The simplest method in this class is pulse code modulation, **PCM**. Here the incoming signal is sampled, usually at the Nyquist rate, and each individual sampled is independently quantized. For adequate amplitude resolution 6-8 bits/pel are necessary. For less than 6 bits/pel a contouring effect becomes visible in the low detail regions. These contours may be broken by adding a small amount of pseudo-random noise to the luminance samples prior to quantization, and subtracting the same noise at the receiver. Alternative methods of combating conoturing are known e.g. coarse fine quantization [5].

In PCM the conversion of continuous to discrete quantitities occurs in a memoryless fashion. Schreiber [7] has estimated, that for image data the first order entropy, $H[x]$, is 4.4 bits/pel whereas the second order entropy, $H[x|x-1]$, is

only 1.9 bits/pel. By exploiting the redundancy present in images, considerable improvement in compression can be achieved. ***Predictive techniques*** achieve this by decorrelating the succesive samples to be quantized. Here the sampled image, $x_{i,j}$, is represented in terms of another array, $e_{i,j}$, formed by subtracting the predicted value of $x_{i,j}$ from $x_{i,j}$, which has no redundancy and from which $x_{i,j}$ can be determined uniquely. Techniques such as ***DPCM*** try to minimize the distortion that results in quantizing $e_{i,j}$ rather than $x_{i,j}$ [2,5]. Distortion measures to be minimized may be the minimum mean square error criterion or certain more complex functions based on subjective or psychovisual fidelity such as contrast or visibility. The latter result in subjectively better images but are difficult to implement [2,3,4,8].

The simplest predictive scheme is a ***delta modulation*** scheme. This uses a one bit quantizer. The predictor simply performs an integration of the quantizer output signal, which is a sequence of binary pulses. An ordinary delta modulation scheme suffers from slope overload errors, granularity noise and instability to transmission errors [5].

The ***DPCM*** scheme is relatively free of the ills that plague the delta modulation.The DPCM scheme is illustrated in fig. 2.1.The predictor here takes a linear combination of previously reconstructed values to form the estimate of a particular pixel, and the error is quantized, coded (usually to 2-4 bits) and transmitted. The quantizer should minimize the degradations associated with DPCM, viz., granularity, slope overload and edge busyness [5,6]. Predicting S_0 based on S_1, the first order predictor, is simply implemented but suffers from blurring of edges, edge busyness and low tolerance to



transmission errors. The subjective quality of an image and the susceptiblity to channel errors can be improved by two-dimensional prediction. For example, a predictor using S_1 and S_4 for prediction reproduces vertical edges with much less blurring than a predictor using just S_1. Predictive schemes give an average saving of 4-5 bits/pel over 8 bit PCM [2-6]. The limitations of DPCM, apart from its high tolerance to transmission errors, are its sensitvity to variation in image statistics, which result in non-uniform error variances over different regions of an image [2].

To take into account the non-stationarities of image statistics ***adaptive techniques*** may be used [5]. Here the quantizer and/or the predictor is altered on a decision rule based on the local image properties. Adaptive predictors provide a much better subjective quality, especially at the edges. Here the image is segmented into regions and a quantizer is chosen for each region depending on the activity in it. Although these schemes involve a complex implementation, adaptive quantization coupled with a variable length coding scheme achieves a significant reduction in bit rates. Predictive techniques are discussed further in chapter 2.

Other simple spatial domain techniques include ***run length coding*** where the number of bits between successive changes in detail or edges is transmitted, together with the amplitude of the edge. To achieve bandwidth compression, the runs of a picture are gathered and redistributed in a transmission line sequence, at a rate equal to the average rate of their occurance, and transmitted. As an example, for an edge probability of 0.1, an average rate of 1 bit/pel is required compared to 3 bits/pel for PCM.

Facsimile coding can be done using both run length encoding and predictive coding of data. From the type of data to be sent, estimates of run length

distributions are obtained, and Huffman code words are assigned to runs of white or black upto a maximum decided length. Use is made of the fact that run lengths are highly correlated in adjacent lines. An example of a simple predictive scheme in facsimile transmission is to declare the predicted value of a pel as the majority state of the pels in the prediction window. Alternatively, a predictor may be designed to minimize the prediction error probability [2].

In broadcast television large redundancies exist between successive frames. An effective ***interframe coding*** technique, that combats errors like aliasing, contouring, and flickering, is the interlace method used in broadcast TV. This brings down the data rate by a factor of two, but requires the storage of half a frame at the receiver. Since the pel values in successive frames change only in areas of relative motion, ***predictive methods*** can be used here, and can be simple to implement compared with other methods. The ***conditional replenishment*** updates only pel values where substantial motion is discerned [5]. It has also been found that spatial resolution in areas of relative motion and temporal resolution in stationary areas may be sacrificed without loss of fidelity. This property is utilized to achieve further compression. In areas of relative motion temporal prediction is poor and spatial prediction is used. Schemes with adequate buffer control have been suggested [2,6]. More complex schemes classify the regions in successsive frames as stationary, slow or fast, and appropriately update the pel values by using a different predictor and/or quantizer in each case. ***Motion interpolation and compensation mehods*** try to predict the motion trajectories of each pel, and then propagate each pel along this trajectory in successive frames, which are not transmitted, till a new reference frame is reached. Details of displacement estimation techniques can be found in [4].

2) Spatial processing image coding

The methods in. this family include transform coding techniques, interpolative methods, SVD (singular value decomposition) coding and feature coding methods.

In ***source interpolative schemes***, the luminance values of an image are approximated by continous functions within some continuous error band. Parameters describing these functions and the range over which they span the pixels is transmitted. The interpolation is usually limited to zeroth and first order polynomials since fitting higher order polynomials, like cubic splines, entails complexity. In ***destination interpolation*** the image is prefiltered and sampled at a rate below the Nyquist rate and the interpolation at the receiver is performed using sinc or bessel waveforms [5].

The method of ***transform coding*** yields impressive compression ratios. The basic premise of an image transform coding system is that in a two dimensional unitary transform of an image, nearly all the information is packed into a small number of samples. To achieve bandwidth reduction the low magnitude transform coeffecients can be disregarded without introducing serious image degradation. In optimum predictive coding successive inputs to the quantizer are whitened, and in general, the optimum predictor is a non-linear causal filter. Performing a unitary transform on an image implies that all the samples are uncorrelated jointly and then quantized. The ***Karhunen-Loeve transform*** represents the image vector in a space whose basis vectors are the eigen-vectors of the covariance matrix of the image. This transform has the property that it packs more energy into a given number of samples than any other transform. Because of this reason, the K-L

transform is best suited to transform coding but it has the drawback that the statistical knowledge it requires is seldom available. Also, it does not possess a fast algorithm. It has been shown that the ***cosine transform*** approaches the efficiency of the K-L transform for Markov process data, and also has a fast algorithm. These reasons explain the extensive popularity of the cosine transform in image coding. The ***Hadamard*** and ***Haar Transforms*** are utilized because of their effecient computational algorithms [2-5]. .

In transform coding the coeffecients to be transmitted are selected using either of two methods given below:

1) ***Zonal coding*** where all the coeffecients lying in a certain decided zone in the two dimensional transform domain are coded and transmitted.

2) In ***threshold coding*** the samples selected for transmission are those which exceed a chosen threshold.

The number of quantization levels are usually proportional to the estimated variance of the component and the number of code bits to its expected probability of occurance. New approaches try to adapt the quantizer to the characteristics of the human visual perception rather than use the m.m.s.e. criterion. In threshold coding the additional brunt of the locations of the samples being transmitted must also be borne. Spectrum extrapolation techniques may be used in conjunction with transform coding. Here the receiver forms the estimates of the transform coeffecients that are not transmitted, before taking the inverse transform. In general transform coding gives much better compaction than predictive coding, especially at low distortion levels [2] and when the image data is non stationary. It also has a better transmission error tolerance, since the errors are distributed over the entire image. But it has the disadvantage of a formidable implementation

complexity.

Adaptive transform coding tries to control the quantizing and coding of individual coeffecients. as a function of picture content or spectral energy such that the resulting distortion is kept below a visibility threshold fixed by a human observer. Another attempt to exploit the properties of human vision for transform coding is the ***M-transform*** [4]. This transform uses the basis functions of a noise-like structure and generates quantization error patterns which are less visible than those of other transforms.

Hybrid coding schemes have been proposed which possess many attributes of transform coding, but are simpler to implement. Here a one dimensional transform is taken along each of the rows of the image block. Each transform coeffecient sequence is then coded independently by a one dimensional predictive technique such as DPCM in the orthogonal direction. Consider the jth row of an image u_j and its transform v_j. Now DPCM is employed to encode the transform coeffecients in the following manner:

$$v_j(i) = \rho_j \, v_{j-1}(i) + e_j(i) \quad , \quad 1 \leq i \leq N$$

where $v_j(i)$ is the ith compnent of the vector v_j and $e_j(i)$ is the quantizer input. Hybrid schemes give good performance upto rates 1 bit/pel.

All the above schemes have the common property that the conversion of continuous quantities to discrete quantities is done on scalars. A fundamental result of rate distortion theory, the branch of information theory that deals with data compression, is that better performance can always be achieved bycoding vectors instead of scalars, even when the data source is memoryless. ***Vector Quantization systems*** map a sequence of continuous or discrete vectors into a digital sequence suitable for transmission. In VQ systems, the collection of all

possible reproduction vectors is called a *codebook*. Starting with an initial codebook, the Lloyd's algorithm, generalized now for vectors, can be iteratively employed on a training sequence of vectors to obtain a quantizer for vectors, which maps all the vectors lying in a particular region to a digital sequence. This quantizer is optimal in the sense that some distortion measure is minimized for a particular family of vectors. Analogs of predictive coding have been proposed for VQ systems with memory [9].

Certain recent advancements, called ***second generation coding methods*** [8], make extensive use of the advancements in the brain mechanisms of vision to achieve higher compressions than the methods discussed above. Typically, they yield compression ratios of 50:1 as compared to 15:1 possible using transform coding. These methods are likely to achieve even better compressions than that predicted by the entropy of popular image models. This is because firstly these conventional models are simplistic, and secondly because they do not take into account what the eye sees and how it does so. In the discussion that follows the model for the eye is assumed to be a LPF representing the spherical aberration, a logarithmic nonlinearity representing the photoreceptor response, and a HPF representing the lateral inhibition process in the eye [10].

The ***local opertor based methods*** are developed on the ***synthetic high method or sub-band coding*** which has been known for a long time. In this scheme [5,8], the image is first split into low and high frequency components. The low pass image can be sub-sampled, in accordance with the sampling theorem, and about 5 bits/pel give adequate resolutions. The high frequency components can, in turn be coded using about 3 bits/pel because the eye can tolerate larger brightness errors in this frequency range because of its low sensitvity in this frequency range. In the ***pyramidal coding*** method the image $x_{i,j}$ is first low pass filtered to give $x_{i,j}^{1}$ and

the error image

$$e_{i,j}^{1}=x_{i,j} - x_{i,j}^{1}$$

Coding the error and the low pass filtered image, instead of the original image is economical, as explained above. The above process is repeated a number of times to give

$$e_{i,j}^{k}=x_{i,j}^{k} - x_{i,j}^{k+1} =x_{i,j}*(h_{i,j}^{k} - h_{i,j}^{k+1})$$

The difference of two gaussian like LPF's, h^k and h^{k+1} , is good approximation to the lateral inhibtory response of the eye. Note that the information needed to code x^{k+1} is less than that required to code x^k because of the sampling theorem. Good quality pictures with average compressions of 10:1 are obtained using the pyramidal coding scheme.

The ***anisotropic non-stationary predictive coding scheme*** is another hybrid coding scheme which yields high compactions, typically 50:1 without undue sacrifice of picture quality. This makes use of the non-isotropic Wiener filter, traditionally used for image restoration in the presence of additive, white noise, as a prediction filter. The high compressions in this framework are due to the fact that the filter makes use of parameters related to the magnitudes and directions of local rectilinear features in the image and this may be viewed as the estimation of non-linearities within the image. Details may be found in [8].

The second group within the second generation methods achieves economy by representing images in terms of textured regions surrounded by contours, in such a way that the regions correspond to objects in the scene. The contours and textures are coded separately. Effecient codes exist for contour coding. Because of the low information content texture can also be coded effeciently. Contours may be detected in two ways: by region growing and by edge detection. In region

growing based methods the image is iteratively fitered by an *inverse gradient filter*, which removes the granularity in an image without affecting the contours. The mechanism of region growing is as follows: regions are first characterized by some property e.g. the gray level of a pel. Starting with a pixel, the succeeding pixels are examined to find if they have the same property as the first pel. When the process is over, a large number of contours are obtained that do not necessarily correspond to objects in the scene. The number of contours is reduced by eliminating those regions which contain very few pels in them, those regions in which the contrast difference from the surrounding areas is below a threshold and also removing those contours which are contained wholly inside others. The remaining contours are then approximated by line segments, arcs of circles etc., and this information is coded. After this, the general gray level shape in the regions described by the contours is appproximated by a two-dimensional polynomial function. The granularity is added in terms of pseudo-random noise and the polynomial coeffecients are transmitted. Compression of about 50:1 are attained in this way. Details of this method and others, like directional decomposition based coding, may be found in [8].

After this brief survey of the practices prevelant in image coding, the problem considered in this work is formulated.

1.2 The problem

The need often arises to reevaluate and modify traditonal methods in order to achieve certain ends in a better way. Over the years, the AR, MA and ARMA models have established their stronghold over the field of image modelling. However, in [11] it is shown that these linear models are nearly gaussian. Since image data is

mostly non-gaussian, alternative models for non-gaussian time series need to be investigated. Numerous such models, directed towards representing non-gaussian time series have been constructed and studied by various authors [1,14-20]. The above work extends the standard AR, MA and ARMA schemes to processes, the variants of which may prove useful fo image modelling.

More important than the fact that the processes are represented by a non-gaussian model, is the particular structure of these processes which makes them amenable to more compact representation of images, than is achievable through conventional predictive methods like the DPCM. One such scheme which shows promise in image coding has been suggested by Gibson [1]. This is based on the first order AR process in which the excitation is absent with a probability p. The data compression of such processes has never been investigated before. The objective of this study is to evaluate the performance of this scheme in terms of implementation feasibility, the compressions achieved and the degradation caused in coding various real life imagery. Attention is also paid to identifying the best methods among a plethora of possibilities, investigation into the use of similar other models suggested in literature and the extensions of this scheme to higher orders and dimensions.

1.3 Organization of the report

The report on the study is organized as follows:

Chapter 2 introduces the properties of the IEAR(1) process and its variants. The method of choosing the predictor coeffecient, the compression promised by this scheme and its differences with the traditional AR processes are explored. The DPCM and IEAR(1) coding methodologies are

contrasted.

Chapter 3 considers the details involved in the implementation of the algorithm for image coding using the IEAR(1)/IEAR-IC(1) process and catalogues the various configurations in which these may be used for image compression purposes.

Chapter 4 discusses the results obtained in coding real life imagery using the IEAR(1) process and its variants. These are compared with those obtained in image coding using 3-bit DPCM. The configurations of the algorithm which yield the best overall performance are identified. It finally reviews the overall conclusions reached in the study and points out the directions for further research.

Chapter 2

The IEAR(1) Process

This chapter discusses the IEAR(1) process within a general framework of predictive coding methods.

2.1 Predictive techniques for image data compression

In pulse code modulation (PCM), successive inputs to the quantizer are treated in a memoryless fashion. Often the data to be transmitted has statistical redundancy from one sample to the next. Consider a random sequence $\{x_n\}$ in which the the samples upto n=k-1 have been transmitted somehow. Let x^R_n denote the reproduced value of x_n. In predictive schemes, when x_k is to be transmitted, a quantity $\hat{x}^R_k$ is predicted from the previously transmitted values and a prediction error sequence

$$e_k = x_k - \hat{x}^R_k \tag{2.1.1}$$

is formed and quantized to give e^Q_k. The reproduced value of x_k is given by

$$x^R_k = \hat{x}^R_k + e^Q_k \tag{2.1.2}$$

A common method using predictive quantization is DPCM. The principal components of a pedictive coding system are the predictor and the quantizer (fig. 2.1).

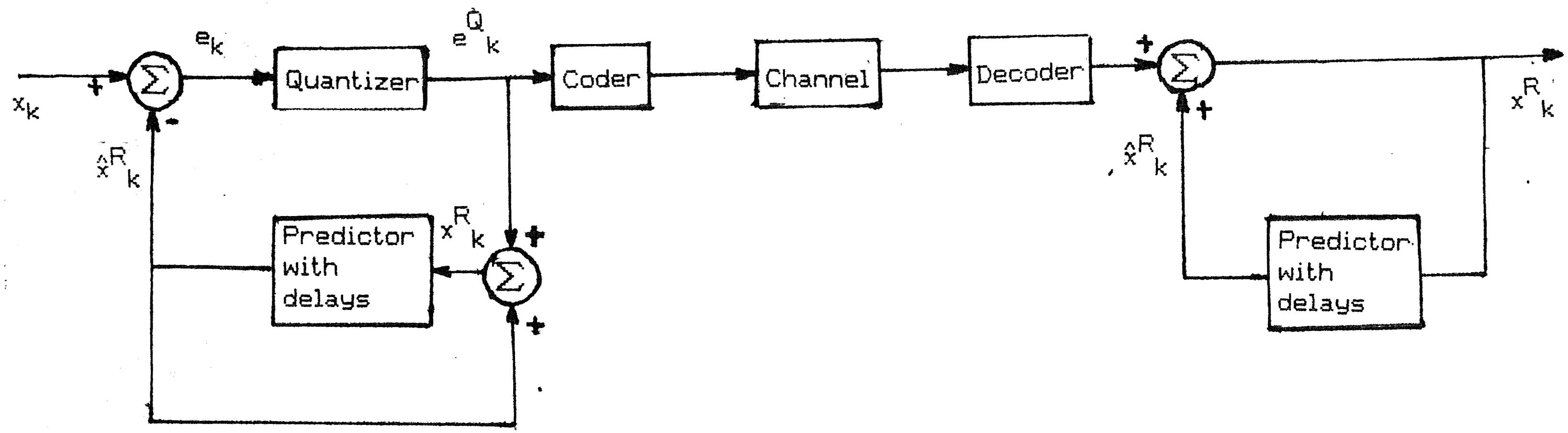


Fig. 2.1 A predictive coding scheme

The error in reproduction of x_k is given by

$$\delta x_k = x_k - x^R_k = e_k - e^Q_k \tag{2.1.3}$$

and is equal to the error in the quantization of e_k.

To minimize $\sigma^2_{e_k}$, the variance of the predicton error, $\hat{x}^R_k$ should be the best mean square estimate of x_k, viz., the mean of the aposteriori density i.e.

$$\hat{x}^R_k = E\{x_k | X^{R^-}_k\} \tag{2.1.4}$$

where $X^{R^-}_k$ is the set of past reproduced values i.e.

$$X^{R^-}_k = \{x^R_n,\ n<k\}$$

From eqn.(2.1.3) the mean square distortion in x_k is given by

$$E\{\delta x^2_k\} = \sigma^2_{q_k} \tag{2.1.5}$$

what is q_k = $e_k - e^q_k$ = quan[illegible]

From rate distortion theory [12], it is known that the quantization of independent gaussian samples, of variance σ^2, can be done at an average bit rate

$$n = \frac{1}{2}\log_2 \frac{\sigma^2}{D} \tag{2.1.6}$$

where D is the average mean square distortion per sample and $D < \sigma^2$ [2,12].

Assuming e_k to be gaussian, the quantization of e_k can be achieved at the minimum achievable rate

$$n_{DPCM} = \frac{1}{2}\log_2\left(\frac{\sigma^2_{e_k}}{\sigma^2_{q_k}}\right) \tag{2.1.7}$$

If the original sequence were quantized, the minimum achievable rate for x_k would be

$$n_{PCM} = \frac{1}{2}\log_2\left(\frac{\sigma^2_{x_k}}{\sigma^2_{q_k}}\right) \tag{2.1.8}$$

for the same quantizing distortion $\sigma^2_{q(k)}$

Hence the reduction in achievable rate of predictive quantization over DPCM is

$$n_{PCM} - n_{DPCM} = \frac{1}{2}\log_2\left(\frac{\sigma^2_{x_k}}{\sigma^2_{e_k}}\right) \tag{2.1.9}$$

Hence, the data compression ability depends upon the variance reduction by prediction, i.e., the ability to predict x_k and therefore on the intersample dependence of $\{x_n\}$. Therefore, the underlying principle in DPCM is to remove the mutual redundancy between successive samples and to quantize only the new information [2].

The introduction of the quantizer inside the feedback loop presents analytical difficulties. In most treatments, it is convenient to consider the prediction process separately from the quantization procedure.

The next section discusses some properties of the standard AR process. This will be followed by a brief note on the DPCM of these processes.

2.2 Autoregressive processes

An image is often considered as a collection of one-dimensional signals. The one-dimensional signal is obtained from the two-dimensional image field by a suitable concatenating strategy, e.g., by considering the samples to be the output of a raster scanner. In the AR representation, the one-dimensional signal is assumed to be generated by a linear system, whose impulse response is specified by a difference equation, and which is forced by a white noise or another random sequence of known spectral density function (abbreviated SDF) [2, 22].

The pth order causal AR representation for a zero mean, stationary random sequence is given by

$$x_k=\sum_{n=1}^{p} a_n x_{k-n}+\epsilon_k\,, \qquad E\{\epsilon_k\}=0,\ E\{\epsilon_k\epsilon_l\}=\beta^2\delta_{k,l} \tag{2.2.1}$$

where ϵ_k is a zero mean, white noise, independent of x_l for $l < k$.

For a stationary process with known autocorrelations, the coeffecients a_n can be determined by solving the well known Yule-Walker equations, using Levinson's

algorithm [2]. Once the coeffecients a_n have been so determined, the quantity

$$\hat{x}_k=\sum_{n=1}^{p} a_n x_{k-n} \tag{2.2.2}$$

is the best mean square predictor of x_k based on the past p values.The sequence x_k, defined by ~~(2.10)~~ (2.2.1) above, is a pth order Markov process. The AR process is asymptotically stationary and stable, in the BIBO sense, if the roots of the polynomial

$$A_p(z)= 1-\sum_{n=1}^{p} a_n z^{-n}$$

lie inside the unit circle. If the process is stationary it's SDF is given by

$$S_x(z)= \frac{\beta^2}{A_p(z)A_p(z^{-1})} \quad , z=e^{j\omega}, -\pi<\omega<\pi \tag{2.2.3}$$

Given an arbitrary set of positive definite covariances $\{r_k\}$ on a window $W:\{-p \le k \le p\}$, there exists a unique AR model which can generate these r_k over W. An important consequence of this result is that any given SDF can be approximated arbitrarily closely by a finite order AR spectrum, i.e.,

$$\underset{p\mapsto\infty}{\mathrm{Lt}} \; \frac{\beta_p^2}{A_p(z)A_p(z^{-1})}=S_x(z) \tag{2.2.4}$$

Moreover, outside this window W, there exists a unique maximum entropy extrapolation of the given covariance sequence, and is given by the covariances generated by the AR model of ~~(2.10)~~ (2.2.1)

$$r_{n+p}=\sum_{k=1}^{p} a_k r_{n+p-k} \quad , \quad n\ge 1. \tag{2.2.5}$$

If the order of the AR process is large, x_k is the sum of a large number of random variables and hence, the density of x_k is the convolution of the densities of the random variables on the right side of ~~(2.10)~~ (2.2.1). It follows from the central limit theorem that x_k is gaussian as $p\mapsto\infty$.

The properties of the AR process dicussed above will be useful in

comparison between the AR and the IEAR processes.

2.3 DPCM of markov processes

If the signal is modelled to be a pth order Markov process

$$x_k = \sum_{n=1}^{p} a_n x_{k-n} + \epsilon_k \tag{2.3.1}$$

the best predictor for x_k in the m.s. sense is

$$\hat{x}_k = E\{x_k | x_l \; , \; k-p \le l \le k-1\} = \sum_{n=1}^{p} a_n x_{k-n} \tag{2.3.2}$$

Hence in DPCM the predicted value is taken to be

$$\hat{x}^R{}_k = \sum_{n=1}^{p} a_n \, x^R{}_{k-n} \tag{2.3.3}$$

An encoder-decoder may now be designed to implement this rule.

2.4 The general linear predicton formulation

The general problem for linear prediction in the absence of quantization errors is now formulated.

Suppose the kth sample is to be predicted by taking a linear combination of the past p samples. Let

$$X^T{}_k = [x_{k-1}, x_{k-2}, \ldots\ldots, x_{k-p}], \tag{2.4.1}$$

$$A^T = [a_1, a_2, \ldots\ldots\ldots\ldots, a_p] \tag{2.4.2}$$

and $\hat{x}_k = A^T X_k$ is the predicted value. (2.4.3)

The prediction error is

$$e_k = x_k - \hat{x}_k = x_k - A^T X_k. \tag{2.4.4}$$

Now, $$e^2{}_k = x^2{}_k + A^T X_k X^T{}_k A - 2x_k X^T{}_k A \tag{2.4.5}$$

and $$E\{ e^2{}_k\} = E\{ x^2{}_k\} + A^T E\{ X_k X^T{}_k\} A - 2E\{ x_k X^T{}_k\} A$$

$$= R_x(0) + A^T \mathbf{R_x} A - 2P^T A \tag{2.4.6}$$

where $\mathbf{R_x}$ is the (p-1) x (p-1) correlation matrix for $\{ x_i\}$

and $\qquad P^T = E\{ x_k X^T_k\} = [R_X(1), R_X(2), \ldots\ldots, R_X(p-1)]$

For finding the weight vector A which yields the minimum m.s.e. we must have $\nabla_A\{E\{e^2_k\}\}=0$, where $\dot{\nabla}_A$ represents the gradient operator. Solving this set of equations we have

$$R_X A = P \quad \text{or} \quad A_{optimum} = R^{-1}_X P \tag{2.4.7}$$

This gives the weight vector which yields the m.m.s.e.. The value of this m.m.s.e. is

$$(E\{e^2_k\})_{min} = R_X(0) - P^T A_{optimum} \tag{2.4.8}$$

It can be shown that for $A=A_{optimum}$, $E\{e_k X_k\}=0$,i.e., the error sequence is orthogonal to the past inputs. Note that these assumptions are only approximately true in the presence of quantization errors. Assuming the process to be ergodic, the autocorrelations may be estimated by finding the time averages. There are two popular methods for doing this, the *autocorrelation method* and the *covariance method*. The equation (2.4.7) above can be written as

$$\sum_{k=1}^{p} a_k R_X(k-i) = R_X(i) \;, \qquad 1 \le i \le p \tag{2.4.9}$$

In terms of the estimates of the a.c.'s, to be obtained from an ergodic time series, this can be rewritten as

$$\sum_{k=1}^{p} a_k \sum_n x_{n-k} x_{n-i} = \sum_n x_n x_{n-i} \;, \qquad 1 \le i \le p \tag{2.4.10}$$

The autocorrelation method assumes the data to be zero outside a window $0 \le n \le N-1$. For this range of n, we have

$$R_X(i) = \frac{1}{N-1-i} \sum_{n=0}^{N-1-i} x_n x_{n+i} \tag{2.4.11}$$

In the covariance method,the error is minimized over a finite interval $0 \le n \le N-1$.The method gives the set of equations

$$\sum_{k=1}^{p} a_k \varphi_X(k,i) = \varphi_X(0,i) \;, \qquad 1 \le i \le p,$$

where $\qquad \varphi_X(i,k) = \sum_{n=0}^{N-1} x_{n-i} x_{n-k}$ $\hfill (2.4.12)$

Details of these two methods can be found in [21]. In [21] it is also shown that linear prediction can be viewed as a technique for autocorrelation or specrum approximation. (See equation (2.2.4) also).

2.5 Image Modelling

Image models are attempts to describe the intensity distribution of a given image. Each model can only capture some important aspect of the intensity distribution and no model can imbibe in itself all the important aspects of the data. For every basic image processing task, such as segmentation or coding, there is a plethora of algorithms and each algorithm assumes a particular model for the image. The performance of different algorithms can be compared theoretically when we understand the underlying image models and the discuss the validity of the assumptions made therein. These considerations help us in understanding the causes behind the success or the failure of a particular algorithm.

The methodology of choosing any model for a given image contains four steps. First, recognition of the type of model appropriate to the given application, recognizing the inevitabilty of noise. The second step is a criterion for estimating the model parameters using the given image. The third is the development of a decision rule for the task, e.g., edge detection. The last step is a qualitative and quantitative analaysis of the performance of the algorithm.

2.6 Alternative AR models

Most of the stationary time series analysis, for statistical convinience, assumes the marginal distribution of the time series to be gaussian. Marginal distribution is not considered of interest per se and the statistical procedures

used very definitely make normality assumptions. For ARMA processes, if the marginal distribution is specified to be gaussian then the innovation sequence is known to be gaussian, also. If some other marginal distribution is specified for the ARMA process the choice of the distribution of the innovation sequence is by no means obvious. Alternative AR, MA and ARMA were constructed [1,14-17] to model data which are known to be non-gaussian. For example, in 1980 Gaver and Lewis determined that for $\{x_i\}$, having a first order AR dependence, to have an exponential distribution with parameter λ, x_k must satisfy

$$\begin{aligned} x_k &= \rho x_{k-1} && \text{w.p. } \rho \\ &= \rho x_{k-1}+e_k && \text{w.p. } 1-\rho \end{aligned} \qquad (2.6.1)$$

where $\{e_i\}$ is an i.i.d. sequence of $\exp(\lambda)$ r.v.'s

This methodolgy was extended to higher order exponential AR, MA and ARMA processes in [15]. Here the process $\{x_i\}$ is exponential, the innovations sequence $\{e_i\}$ is also exponential and the autocorrelations are determined by linear difference equations. The models are linear in both the variables and the parameters, but the linearity is associated with a probabilistic choice between several linear combinations of r.v.'s. Variants of these processes are presented in [16,17].

The following disscusion concerns construction of first order AR models for processes with a specified marginal distribution. These are precisely those models which find use in image coding. Higher order AR, processes are found to be complex to implement and without accompanying increase in compressions or betterment in image quality.

Consider the first order AR process

$$x_k = \alpha x_{k-1}+\epsilon_k \qquad (2.6.2)$$

used very definitely make normality assumptions. For ARMA processes, if the marginal distribution is specified to be gaussian then the innovation sequence is known to be gaussian, also. If some other marginal distribution is specified for the ARMA process the choice of the distribution of the innovation sequence is by no means obvious. Alternative AR, MA and ARMA were constructed [1,14-17] to model data which are known to be non-gaussian. For example, in 1980 Gaver and Lewis determined that for $\{x_i\}$, having a first order AR dependence, to have an exponential distribution with parameter λ, x_k must satisfy

$$\begin{aligned} x_k &= \rho x_{k-1} && \text{w.p. } \rho \\ &= \rho x_{k-1}+e_k && \text{w.p. } 1-\rho \end{aligned} \tag{2.6.1}$$

where $\{e_i\}$ is an i.i.d. sequence of $\exp(\lambda)$ r.v.'s

This methodolgy was extended to higher order exponential AR, MA and ARMA processes in [15]. Here the process $\{x_i\}$ is exponential, the innovations sequence $\{e_i\}$ is also exponential and the autocorrelations are determined by linear difference equations. The models are linear in both the variables and the parameters, but the linearity is associated with a probabilistic choice between several linear combinations of r.v.'s. Variants of these processes are presented in [16,17].

The following disscusion concerns construction of first order AR models for prosesses with a specified marginal distribution. These are precisely those models which find use in image coding. Higher order AR, processes are found to be complex to implement and without accompanying increase in compressions or betterment in image quality.

Consider the first order AR process

$$x_k = \alpha x_{k-1}+\epsilon_k \tag{2.6.2}$$

where ε_l is independent of x_j for l>j.

This process has a dependence structure characterized by an exponentially decreasing autocorrelation function

$$R_x(m)= E\{x_n x_{n+m}\}= E\{x^2_0\}\alpha^m \tag{2.6.3}$$

The characteristic function of the r.v. x_k is

$$\begin{aligned}\phi_{x_k}(\omega) &= E\{\exp[-j\omega x_k]\} \\ &= E\{\exp[-j\omega(\alpha x_{k-1}+\varepsilon_k)]\}\end{aligned} \tag{2.6.4}$$

Because ε_k is independent of x_{k-1}, we have

$$\phi_{x_k}(\omega)= \phi_{x_{k-1}}(\alpha\omega)\phi_{\varepsilon_k}(\omega) \tag{2.6.5}$$

Assuming x_k is marginally stationary, the subscript k may be dropped and we have

$$\phi_\varepsilon(\omega)= \frac{\phi_x(\omega)}{\phi_x(\alpha\omega)} \tag{2.6.6}$$

Now we can specify the distribution of x_k and examine the conditions on the distribution of ε_k which give rise to this distribution of x_k.

If we let the distribution of x_k to be Laplacian, with parameter λ then

$$\phi_x(\omega)= \frac{1}{1+[\frac{\omega}{\lambda}]^2} \tag{2.6.7}$$

From (2.6.6) we find that

$$\phi_\varepsilon= \alpha^2+(1-\alpha^2)[\frac{1}{1+[\frac{\omega}{\lambda}]^2}] \tag{2.6.8}$$

Hence, we conclude that ε_k takes the value 0 with probability α^2 and is a Laplacian distributed r.v. with probability $(1-\alpha^2)$. We can now write the difference equation for the generation of $\{x_i\}$ as

$$\begin{aligned}x_k &= \alpha x_{k-1}+\varepsilon_k \\ &= \alpha x_{k-1} \qquad \text{w.p. } \alpha^2 \\ &= \alpha x_{k-1}+e_k \qquad \text{w.p. } (1-\alpha^2)\end{aligned} \tag{2.6.9}$$

where $\{e_i\}$ is an i.i.d. sequence of Laplcian r.v.'s.

However, it must be noted that given the output distribution, the input distribution can be found dirctly, using (2.6.6), but not vice versa. Also not any characteristic function can be substituted into (2.6.6) and yield a quantity that is a characteristic function. In other words, not all output distributions are possible. Let the output distribution be symmetric with a zero mean. We wish to determine the conditions on the characteristic function of the output, $\phi_X(\omega)$, such that it can be generated using the first order AR process. $\phi_X(\omega)$ is a real, symmetric, positive definite function equalling unity at the origin. If $\phi_X(\omega)$ is not a decreasing function for positive ω, the ratio $\phi_X(\omega)/\phi_X(\alpha\omega)$ will exceed unity for α greater than some critical value, α_c. In this case, a highly dependent series for a specified marginal distribution would not be possible, whereas one with a lesser correlation would be. Only when $\phi_X(\omega)$ is a decreasing function are all values of alpha permitted. Now, depending on the behavior of $\phi_X(\omega)/\phi_X(\alpha\omega)$ as $\omega\mapsto\infty$, two structures arise. In the first case the tails of the characterestic functions fall sufficiently fast so that the ratio approaches zero. The gaussian and the hyperbolic secant densities have this property and in these there is no zero defect, as visible in (2.6.9). On the other hand, if the limit is non zero, then the density of ϵ_k must have an impulse with an area equal to the limiting value at the origin. With a probability equal to this value, the input is zero at a given sample. This is manifested by the Laplacian and the exponential densities.

As another example consider the case when the marginal distribution of x is specified to be uniform over $[-\frac{1}{2}, \frac{1}{2}]$. Then $\phi_X(\omega)= \frac{\sin(\omega/2)}{\omega/2}$. Since $\phi_\epsilon(\omega)= \frac{\phi_X(\omega)}{\phi_X(\alpha\omega)}$, if the numerator is zero at $\omega=\omega_0$ then the denominator equals zero at $\omega= \omega_0/\alpha$. For $\phi_\epsilon(\omega)$ to be bounded at this point ($\omega=\omega_0/\alpha$), the numerator must also equal zero at $\omega=\omega_0/\alpha$ and in turn the denominator must be zero at $\omega= \omega_0/\alpha^2$. This condition then becomes

recursive and hence $\phi_X(\omega)$ must have an infinity of zeros if it has any. Also, only a countably infinite values of α are now permitted. In case of the uniform density these zeros are equally spaced, i.e., $\omega_k=2k\pi$. Here,

$$\phi_\varepsilon(\omega)= \frac{\alpha\sin(\omega/2)}{\sin(\alpha\omega/2)} \tag{2.6.10}$$

For $\phi_\varepsilon(\omega)$ to be a valid characteristic function α must be the reciprocal of any integer greater than one in magnitude.,i.e.,

$$\alpha= \frac{1}{k} \quad ,k=\pm2,\pm3,\ldots\ldots \tag{2.6.11}$$

In this case $\{\varepsilon_i\}$ is an i.i.d. sequence with each element assuming the values

$-(|k|-1)/2|k|$, $-(|k|-3)/2|k|$,........, $(|k|-3)/2|k|$, $(|k|-1)/2|k|$ with equal probability [18-20].

We will call a process represented by

$$\begin{aligned} x_k &= \alpha x_{k-1} && \text{w.p. } p \\ &= \alpha x_{k-1}+ e_k && \text{w.p. } 1-p \end{aligned} \tag{2.6.12}$$

as a ***first order intermittently excited AR process, IEAR(1)***. The IEAR(1) process is completely specified by stating (1) the value of α, (2) the probability p and (3) the distribution of x_k or the distribution of e_k.

A stationary sequence $\{x_i\}$ is said to be time reversible if the joint distributions of $\{x_{n_1}, x_{n_2}, \ldots\ldots, x_{n_k}\}$ and $\{x_{-n_1}, x_{-n_2}, \ldots\ldots, x_{-n_k}\}$ are the same for all k and n_1, n_2, ..., n_k. The sample functions of non-time reversible processes exhibit different statistical properties in the forward and backward directions. All gaussian processes are time reversible since their distributions are determined by the covariance function which is symmetric. Non-gaussian processes may or may not be time-reversible. It can be shown [20] that all non-gaussian processes which are modelled as the outputs of linear systems , in general, and the IEAR(1) process, in particular, are time irreversible. One of the consequences

of non time reversibility of the IEAR(1) process is that the backward mean square prediction error is less than the forward mean square prediction error. As an example consider the case for the uniform distribution of $\{x_i\}$ considered above.

Here, $x_n = \frac{1}{k} x_{n-1} + \epsilon_n$,

with ϵ_n as defined above. In this case x_{n-1} is entirely determined by x_n, i.e.,

$$x_{n-1} = (\ kx_n\ ) \bmod 1 - 0.5 \quad , \quad \text{for } k \text{ even}$$

and $$x_{n-1} = (\ kx_n + 0.5\ ) \bmod 1 - 0.5 \quad , \quad \text{for } k \text{ odd,} \tag{2.6.13}$$

while the error is determining x_n from x_{n-1} is non zero, the mean square error equalling the mean square value of ϵ_n [18-20].

As another example consider the IEAR(1) process described by (2.6.12). For a first order Markov process, all distributions can be generated from the second order distributions. For time-reversibility we require

$$p_{x_{n_1}, x_{n_2}}(x,y) = p_{x_{-n_1}, x_{-n_2}}(x,y)\ , \quad \text{for all } n_1, n_2. \tag{2.6.14}$$

Shifting the axis by n_1+n_2 and using stationarity, we find for time-reversibility

$$p_{x_{n_1}, x_{n_2}}(x,y) = p_{x_{n_2}, x_{n_1}}(x,y)\ , \quad \text{for all } n_1, n_2 \tag{2.6.15}$$

i.e., the second order distribution must be symmetric. Now, all second order distributions can be obtained from $p_{x_{n-1}, x_n}(x,y)$. Hence if $p_{x_{n-1}, x_n}(x,y)$ is symmetric so are all the other second order distributions. Let us see, whether for the process described by (2.6.12), $p_{x_{n-1}, x_n}(x,y)$ is symmetric or not. We have

$$\phi_{x_k}(\omega) = E\{\exp[-j\omega x_k]\}$$

or, $$\phi_{x_k}(\omega) = p\ E\{\exp[-j\omega\alpha x_{k-1}]\} + (1-p)\ E\{\exp[-j\omega(\alpha x_{k-1} + e_k)]\}$$

or, $$\phi_{x_k}(\omega)= p\phi_{x_{k-1}}(\omega) + (1-p)\phi_{x_{k-1}}(\alpha\omega)\phi_{e_k}(\omega) \tag{2.6.16}$$

or, $$\phi_{e_k}(\omega)= \frac{\phi_{x_k}(\omega)-p\phi_{x_{k-1}}(\omega)}{(1-p)\phi_{x_{k-1}}(\alpha\omega)} \tag{2.6.17}$$

Now,

$$\phi_{x_{k-1},x_k}(u,v)= E\{\exp[-jux_{k-1}-jvx_k]\} \tag{2.6.18}$$

or, $$\phi_{x_{k-1},x_k}(u,v)= pE\{\exp[-jux_{k-1}-jv\alpha x_{k-1}]\}+(1-p)E\{\exp[-jux_{k-1}-jv\alpha x_{k-1}-jve_k]\}$$

or, $$\phi_{x_{k-1},x_k}(u,v) = p\phi_{x_{k-1}}(u+\alpha v) + (1-p)\phi_{x_{k-1}}(u+\alpha v)\phi_{e_k}(v) \tag{2.6.19}$$

Substituting $\phi_{e_k}(v)$ from (2.6.17), we have

$$\phi_{x_{k-1},x_k}(u,v) = \phi_{x_{k-1}}(u+\alpha v)\frac{\phi_{x_k}(v)}{\phi_{x_{k-1}}(\alpha v)} \tag{2.6.20}$$

$$\phi_{x_{k-1},x_k}(u,v) = p\phi_{x_{k-1}}(u+\alpha v)+(1-p)\phi_{x_{k-1}}(u+\alpha v)\phi_{e_k}(v) \tag{2.6.19}$$

Now,

$$p_{x_{n-1},x_n}(x,y)=\int_u\int_v \phi_{x_{k-1},x_k}(u,v)\exp[jux+jvy]\ du\ dv \tag{2.6.21}$$

From (2.6.19) we have

$$p_{x_{n-1},x_n}(x,y) =p \int_u\int_v \phi_{x_{k-1}}(u+\alpha v)\ du\ dv+(1-p) \int_u\int_v \phi_{x_{k-1}}(u+\alpha v)\phi_{e_k}(v)\ du\ dv$$

This integral is easily evaluated to give

$$p_{x_{n-1},x_n}(x,y)= p\ p_{x_{n-1}}(x)\delta(y-\alpha x) + (1-p)\ p_{x_{n-1}}(x)\ p_{e_k}(y-\alpha x) \tag{2.6.22}$$

This is clearly asymmetric and this shows that any process described by (2.6.12) is time irreversible.

The autocorrelation function for the IEAR(1) process can be easily determined.

$$R_x(1)=E\{x_kx_{k-1}\}=p\ \alpha E\{x_{k-1}x_{k-1}\} + (1-p)\ \alpha E\{x_{k-1}x_{k-1}\} + (1-p)E\{e_kx_{k-1}\} \tag{2.6.23}$$

Noting that e_k is independent of x_{k-1} for $1 > 0$ and that $E\{e_k\}=0$, the last term

equals zero. Therefore,

$$R_x(l) = \alpha\, R_x(l-1) \tag{2.6.24}$$

or, $$R_x(l) = \alpha^{l}\, R_x(0) \tag{2.6.25}$$

Hence the autocorrelation structure of this process is the same as the conventional AR process.

2.7 Coding of the IEAR(1) process

The structure of the IEAR(1) process makes it conducive to data compression efforts. The equation (2.6.12) is reproduced below

$$\begin{aligned} x_k &= \alpha x_{k-1} && \text{w.p. } p && \ldots 1 \\ &= \alpha x_{k-1} + e_k && \text{w.p. } 1-p && \ldots 2 \end{aligned}$$

The case ....1, when the input is absent will be referred to as the zero error (ZE) condition, and the case ....2 as the non zero error (NZE) condition. It is felt that, since the model incorporates an alternative where the error term is zero, this property would be made use of in simulating solid gray level areas or slowly changing areas better.

~~A coding scheme that makes use of (2.6.12) is shown in Fig.2.2~~.The predictor
8. generates an estimate of the present sample using

$$\hat{x}^R_k = \alpha x^R_{k-1} \tag{2.7.1}$$

If $x_k = \hat{x}^R_k$, then $e_k = 0$ and the coder communicates the occurance of the ZE condition to the receiver by a short binary word. The reconstructed value at the receiver is

$$x^R_k = \alpha x^R_{k-1} \tag{2.7.2}$$

On the other hand, if x_k is not equal to $\hat{x}^R_k$, then the error

$$e_k = x_k - \hat{x}^R_k \tag{2.7.3}$$

is formed and quantized to a suitable number of bits. The quantized error is coded and transmitted , and the receiver reconstructs the sample as

$$x^R_k = \hat{x}^R_k + e^Q_k \tag{2.7.4}$$

Since the number of bits used to code the ZE condition is much lower than the number used to code the quantized error, a reduction in bit rate, for the same SNR, is achieved over conventional DPCM coding, which would assign the same number of bits to code the error, irrespective of the ZE condition. As the probability p increases the compression improves. Further, if the probability p is close to unity, the ZE condition would occur with a high frequency, and the coder could signal a succession of ZE conditions collectively using run length encoding methods, bringing the bit rate down further.

Instead of considering the scheme as one for reducing the bit rate, we may view it one for increasing the SNR, at a given rate. Since less number of bits are allocated to code the ZE condition, more bits can be assigned to a larger number of quantization levels to which the error can be quantized. The fidelity of the reconstructed signal would, in this case, be much better as compared with a DPCM for an equal bit rate. As an example, for p=0.8 and m_{ZE}=0.5 (i.e. an average of 0.5 bits are used to code every ZE condition) and at a rate of 2 bits/sample, m_{NZE}=8 bits/sample, i.e., as many as 2^8 levels can be used to quantize the non-zero error, at a rate 2 bits/sample.DPCM would, on the other hand, would assign 2 bits for every sample and would consequently obtain much lower SNR's.

2.8 Image modelling and coding using the IEAR(1) process

As explained above, the IEAR(1) process is completely specified by the values of α, p and the distributions of either $\{x_i\}$ or $\{e_i\}$. In case of image modelling, one is interested in the distribution of pixel intensities, i.e., the

distribution of $\{x_i\}$. The actual distributions of the pixel amplitudes is very image dependent and can vary from gaussian, uniform, rayleigh to various bimodal distributions. However, restricting the image class under consideration, such as, high altitude photographs of the earth or head or shoulder images, reasonable assumptions on the pixel amplitude distributions may be made. Another point of interest is that when we are trying to generate a process with certain properties, we do not have the complete freedom in choosing the various characteristics and parameters. For example, the sequence $\{e_i\}$ is not Laplacian with $\{x_i\}$ Laplacian, when p is not equal to α^2 [1]. Previous experiments with the differential encoding of images have shown that images with widely varying distibutions of pixel amplitudes have a pixel amplitudes which are Laplacian distributed. Hence, it is difficult to decide whether to fix the distribution of $\{x_i\}$ or to fix the distribution of $\{e_i\}$ in equation (2.6.12).

The difficulties in completely specifying an IEAR(1) model for a given region of the image does not preclude the use of the model in image coding. This follows because the precise distributions of either $\{e_i\}$ or $\{x_i\}$ find no use in the coding of the IEAR(1) process, using a differential encoding scheme, as delineated in the last section. In the coding of an image using an IEAR(1) process, an attempt is made to identify a particular region of an image, obtained after suitably processing the image to get a one-dimensional signal, with an IEAR(1) process. This is achieved by forming the sequence

$$y_k = \frac{x_k}{x_{k-1}} \tag{2.8.1}$$

and assuming p, the probability of the ZE condition, to be ≥ 0.5. Then from (2.6.12), we have

$$\begin{aligned} y_k = \frac{x_k}{x_{k-1}} &= \alpha && \text{w.p. } p \\ &= \alpha + \frac{e_k}{x_{k-1}} && \text{w.p. } 1-p \end{aligned} \tag{2.8.2}$$

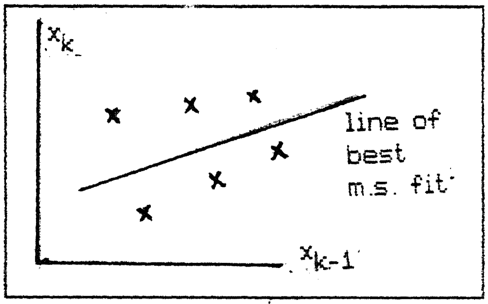
Hence, the sequence y_k is examined, for each region of the derived 1-d signal, and a recurrence of equal values of y_k implies that this particular region can be identified with an IEAR(1) process with prediction coeffecient α. The value of p and the distribution of $\{e_i\}$ are left unspecified, and may be assumed to be either changing over the given region or as functions too complicated to be described exactly. The coding is achieved by the method outlined in section 2.7. Over another region a different IEAR(1) model may be valid. In case the recurrence of equal values of y_k does not happen, over a particular region, this region cannot be taken to be the sample function of any IEAR(1) process. It is shown in sec. 3.1 that to achieve economy, over such regions it may be economical to revert to DPCM for coding such regions. Note that the assumption of ergodicity is inherent in the above approach since the model parameters are being estimated from the sample functions themselves.

The above approach tries to isolates regions of stationarity in a one dimensional signal derived from the image, and each such region is the sample function of a different IEAR(1) process. This approach is essentially adaptive, since the predictor is altered as the statistics of the input process change. A simpler approach would be a non-adaptive one, where a value of α is decided a priori, depending upon some assumptions about the image statistics, and this value of the predictor coeffecient is used for the entire image. Here the entire image is being considered as the realization of a single IEAR(1) process, or a concatenation of IEAR(1) processes, with the same value of α but with the probability p and the distribution of e_k that change over the image. The performance of the two schemes, the adaptive and the non-adaptive are compared later.

2.9 Differences of the IEAR(1) process and the inbuilt coding methodology with the conventional AR processes and the DPCM

Consider a first order predictor for DPCM. The quantization errors are considered negligible. The prediction error is given by

$$x_k = \alpha' x_{k-1} + \epsilon_k \tag{2.9.1}$$



The value of the prediction coeffecient α' that minimizes the mean square error, $E(\epsilon^2_k)$, is given by

$$\alpha'_{opt} = \rho(1) = \frac{R_x(1)}{R_x(0)}$$

Now consider the IEAR(1) process

$$\begin{aligned} x_k &= \alpha x_{k-1} && \text{w.p. } p \\ &= \alpha x_{k-1} + e_k && \text{w.p. } 1-p \end{aligned}$$

The mean square error is given by

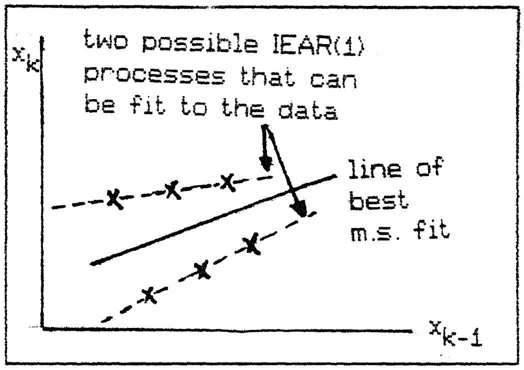
$$E(e^2_k) = p.0 + (1-p).E((x_k - \alpha x_{k-1})^2) \tag{2.9.2}$$

And the value of α that minimizes the m.s.e., $E(e^2_k)$, is again

$$\alpha_{opt} = \rho(1) = \frac{R_x(1)}{R_x(0)} \tag{2.9.3}$$

Suppose an IEAR(1) process is generated with a known α (and some value of p and some distibution of $\{e_i\}$). For this process, the autocorrelation function is known and we must have $R_x(1) = \alpha\ R_x(0)$. But $\alpha = \alpha_{opt} = R_x(1)/R_x(0)$ gives the slope of the line with the best m.s. fit in the x_k-x_{k-1} plane. Also this line must pass through at least half the points in the x_k-x_{k-1} plane, for $p \geq 0.5$. Hence we see that for an IEAR(1) process with known autocorrelations, the best m.s. line passes exactly through at least some of the points in the x_k-x_{k-1} plane. In this case the coding process would inherit all the properties of DPCM, such as orthogonality.

When we fit a model to a given time series, the underlying distributions of which are unknown, the situation is different from the one above. Here, two lines can be fit to the data (asssuming an IEAR(1) representation of the process can be found). The first is a line passing through some of the points in the x_k-x_{k-1} plane, the slope of which gives the value of the predicton coeffecient, α, for the coding procedure corresponding to the associated IEAR(1) process. The other is a line providing the best m.s. fit in the x_k-x_{k-1} plane, associated with DPCM. The prediction coeffecient for DPCM is determined from $\alpha'_k = \frac{\sum_i x_{i-1} x_i}{\sum_i x_i^2}$ where the



range for i may be chosen in accordance with one of the the well known autocorrelaton or the covariance methods. In case of the IEAR(1) process the predicton coeffecient is determined from the recurrence of equal values of $y_k = \frac{x_k}{x_{k-1}}$. Because the line corresponding with the IEAR(1) fit is, in general, different from the line of the best m.s. fit, the IEAR(1) process would result in a larger m.s. prediction error but affords greater compression since the occurance of the ZE conditon facilitates economy. Also because the prediction coeffecients for the two cases are different, for the coding using the IEAR(1) model, orthogonality would not hold, and consequently the complete decorrelation of the error is not achieved, resulting in larger error variances. It is also clear that whereas it is always possible to fit an AR(1) model to a time series it is not always possible to do so in the case of an IEAR(1) model.

2.10 Variants of the IEAR(1) model

In the above discussion the implicit assumption has been $|\alpha|<1$. If $|\alpha|\geq 1$, the filter $x_k = \alpha\, x_{k-1}+ \epsilon_k$ is unstable in the BIBO sense and also, since $R_x(k)=\alpha^k R_x(0)$ this reperesents an unrealistic autocorrelation situation in that the a.c.'s are increasing with the time index, k. But the case of $|\alpha|\geq 1$ is necessary in this development in order to model increasing or constant gray level regions in an image. Also since any image is necessarily of finite dimensions, here we will not bothered by conditions of BIBO stability. Hence we make no distinctions between the cases $|\alpha|<1$ and $|\alpha|\geq 1$. This is another way of saying that, in the case of these algorithms, α no longer has the connotation of a first order autocorrelation.

The IEAR(1) process taken up above enables one to model regions of either just increasing or just decreasing or constant gray level values. To accomodate regions in which gray levels both increase as well as decrease, another process may be constructed, the ***IEAR-IC(1) process*** defined by

$$
\begin{aligned}
x_k &= \alpha x_{k-1} && \text{w.p. } p_1 \\
&= \alpha x_{k-1} + e_k && \text{w.p. } p_2 \\
&= \tfrac{1}{\alpha} x_{k-1} && \text{w.p. } p_3 \\
&= \tfrac{1}{\alpha} x_{k-1} + e'_k && \text{w.p. } p_4
\end{aligned}
\tag{2.10.1}
$$

with $\sum_1^4 p_i=1$ and $|\alpha|<1$. $\{e_i\}$ and $\{e'_i\}$ are independent sequence of r.v's.

The abbreviation *IEAR-IC(1)* then, stands for the *first order intermittently excited AR process with inverse coeffecient*. Note that the prediction coeffecients for the IEAR-IC(1) process can be specified by just the value of α and this is important from the point of view of parsimony. In this case, for a chosen region of the 1-d signal derived from a given image, the sequence y'_k with

$$y'_k=\frac{x_k}{x_{k-1}}$$

$$=\frac{x_{k-1}}{x_k}, \quad \text{if } \frac{x_k}{x_{k-1}}>1 \tag{2.10.2}$$

is formed and the value of α decided depending upon the recurrence of equal values of y'_k. The predictor first forms the estimate

$$\hat{x}^R_k = \alpha x^R_{k-1} \tag{2.10.3}$$

If $x_k=\hat{x}^R_k$, the coder signals this condition to the receiver. Otherwise, the predictor forms the estimate

$$\hat{x}^R_k = \frac{1}{\alpha}x^R_{k-1} \tag{2.10.4}$$

Again if $x_k=\hat{x}^R_k$ then this conditioned is signalled. If $x_k\neq\hat{x}^R_k$ again, then the error $e_k= x_k-\hat{x}^R_k$ is formed, quantized, coded and transmitted. Note that in this case one additional bit is required, over the IEAR(1) process, to signal the ZE condition which may arise in two possible ways. But the frequency of the ZE condition in this case is expected to be higher than that in the case of the IEAR(1) process.

The autocorrelation function of the IEAR-IC(1) process is

$$R_X(l)=\rho^l R_X(0) \quad , \rho= \alpha + (\tfrac{1}{\alpha}-\alpha)(p_3+p_4) \tag{2.10.5}$$

where $p_2=1-p_1-p_3-p_4$ has been used in the derivation.

In general, the coding process for the IEAR-IC(1) model would result in larger error variances than DPCM and does not decorrelate the error fully.

2.11 Higher order and multidimensional processes

Consider the second order process

$$x_k=\alpha_1 x_{k-1} \quad \text{w.p.} \quad p_1$$

$$=\alpha_2 x_{k-2} \quad \text{w.p.} \quad p_2$$

$$=\alpha_3 x_{k-1} + \alpha_4 x_{k-2} \quad \text{w.p.} \quad p_3$$

$$=\beta x_{k-1} + \gamma x_{k-2} + e_k \quad \text{w.p.} \quad p_4 \qquad (2.11.1)$$

The assumptions on $\{e_i\}$ and p_i's are similar to those made in the previous sections. The considerations in the construction of this process are different from those in [15]. Here we must remember that this model will find its use in image coding and we are not specifically interested in a solving a set of equations from which the distribution for the excitation, for a specified marginal distribution of the output, can be determined easily. A coding strategy based on the IEAR processes suffers from an increasing overhead, which comes into play, apart from that involved in quantizing and coding the various coeffecients, as one of an increasing number of possible ZE conditions must be signalled to the receiver, as the order of the IEAR process increases. For example, in the above case, two additional bits will be required to code which of the three possible ZE conditions has actually occured, unless a decision is made to code a succession of ZE conditions, occuring from the same alternative, by run length coding. Moreover, since the philosophy behind choosing the coeffecients is essentially different from that of DPCM, in that no minimization of the m.s. error is performed, these schemes can make no more claims than being ad hoc techniques to achieve data compression, as opposed to the theoretical soundness of DPCM, which enables it to establish more satisfying tasks like autocorrelation or spectrum approximation. In case of DPCM it can be shown that the error power decreases as the order of the prediction increases. Because the coeffecients of IEAR processes are chosen under a different rationale, no analogous statement can be made for the coding schemes based on these models.

Exactly the same remarks as above can be made for two-dimensional IEAR processes by recognizing that equation (2.11.1) can be converted into a 2-d

prediction scheme simply by replacing x_k by $x_{i,j}$, x_{k-1} by $x_{i-1,j}$ and x_{k-2} by $x_{i,j-1}$. Hence in this work attention is restricted to the compression performance of the IEAR(1) and the IEAR-IC(1) processes in image coding.

Chapter 3

Aspects of implementation

In this chapter we take up the tradeoffs and the heuristics involved in a software implementation of an algorithm for image coding based on the IEAR(1) and the IEAR-IC(1) processes. Two methodologies are followed, as explained in sec. 2.8. The first an adaptive one in which the regions within an image are isolated into sample functions of different IEAR/IEAR-IC processes, each characterized by its own predictor coeffecient α; and the second in which the entire image is considered as the realization of a single IEAR/IEAR-IC process or different IEAR/IEAR-IC processes with the same value of α. Comparison is made with 3 bit/sample DPCM. The implementation of the adaptive scheme is considered first, and that of the other would follow immediately.

Recall that the IEAR(1) process is described by

$$\begin{aligned} x_k &= \alpha x_{k-1} && \text{w.p. } p \\ &= \alpha x_{k-1} + e_k && \text{w.p. } 1-p \end{aligned} \qquad (2.6.12)$$

and the IEAR-IC(1) process by

$$\begin{aligned} x_k &= \alpha x_{k-1} && \text{w.p. } p_1 \\ &= \alpha x_{k-1} + e_k && \text{w.p. } p_2 \end{aligned}$$

$$= \frac{1}{\alpha}x_{k-1} \qquad \text{w.p. } p_3$$

$$= \frac{1}{\alpha}x_{k-1}+e'_k \qquad \text{w.p. } p_4. \tag{2.10.1}$$

3.1 The coding protocol

The actual protocol in signalling is taken up first since this would clarify some points in the earlier stages of trying to decide the value of the prediction coeffecient for a particular region. For the IEAR(1) process one bit is used to distinguish between the ZE and the NZE conditions, i.e., if the ZE condition occurs a zero is transmitted; else a one followed by a 3 bit codeword for the quantized error is sent. Thus a run length coding scheme is not used. This aspect is discussed further in sec.3.6. For the IEAR-IC(1) process again one bit is used to discriminate between the ZE and the NZE conditions. If the ZE condition occurs then another bit is used to indicate which of the two possible cases have caused the ZE condition. Note that if α=1.0 then the IEAR-IC(1) process is the same as the IEAR(1) process, and the coding scheme for this case is the same as that for IEAR(1) process.

Another aspect of the coding protocol needs to be taken up. In general, it may not be possible to fit an IEAR process to any given region of the one dimensional signal derived from the image. In such a region, since the ZE condition does not occur often enough no saving, due to the shorter bit word assignment to the ZE condition, is to be expected. Since the NZE conditions require 4 bits/sample to signal information about the quantized error which would ordinarily take 3 bits, it is better to revert to DPCM while coding such regions. Hence over any length of the signal, we have regions which have been coded following the IEAR(1) philosophy alternating with regions coded using DPCM. This switch from IEAR(1) encoding to

DPCM and vice-versa has to be indicated to the receiver by a word of length equal to that used to encode the the NZE condition, viz., 3 bits to signal the switch from DPCM to IEAR(1)/IEAR-IC(1) and 4 bits to indicate the opposite case.

With this foresight we are ready to go through the various steps in the coding algorithm, systematically.

3.2 Converting the image into a 1-d signal

The conversion of the image into a one-dimensional signal may be done in two ways:

(1) treating the rows of the image individually and ignoring the correlations in the orthogonal direction. This is the practice in broadcast TV, where the image is scanned by means of a raster scanner.

(2) dividing the image into blocks of size 4x4 or 8x8 and considering the 1-d signal to be a concatenation of the rows of the individual blocks. This method makes use of the image correlations in both the directions, especially for the smaller 4x4 blocks.

It must also be remembered that the algorithm uses a buffer equal to the length of the 1-d block and a large image size may make it mandatory to choose smaller 1-d blocks.

3.3 Isolating the IEAR(1)/IEAR-IC(1) regions and the determination of the predictor coeffecients

Once the 1-d block containing the PCM values is acquired, the process of isolating the IEAR/IEAR-IC regions begins, and the entire procedure is repeated for each 1-d block. If regions of validity of the IEAR(1) process are to be isolated

then the sequence y_k, defined in sec. 2.8, is constructed; else, when the coding is to be accomplished using the IEAR-IC(1) process the sequence y'_k, defined in sec. 2.10, is formed. The process of recognizing the regions of the derived 1-d signal as sample functions of IEAR(1)/IEAR-IC(1) processes now proceeds in an identical fashion for either case. Hence, in the following discussion any reference to the sequence y_k will imply both y_k as well as y'_k , and will be valid for the IEAR(1) fit and the IEAR-IC(1) fit, respectively.

Now the coder looks for recurrence of equal values of y_k. Here, it must be allowed for the fact that any model that we are trying to fit to the data, will be only approximately true. Therefore, exact equivalence of y_k values is not expected; rather in a region where an incompletely specified IEAR(1) model is approximately valid, the sequence y_k will take values that will lie in some neighbourhood of the model parameter α. This neighbourhood of α, in a region of the image which, at best, closely conforms to the sample function of an IEAR(1) process with known α, can be attained in two possible ways:

The sample values in the region may satisfy the relationship

$$x_k = \alpha x_{k-1} \pm \epsilon \qquad \text{w.p. } p \qquad \text{crit. 1}$$

or they may satisfy

$$x_k = (\alpha \pm \epsilon) x_{k-1} \qquad \text{w.p. } p \qquad \text{crit. 2}$$

where ϵ defines the neighbourhood of α.

These two criteria for identifying IEAR(1) regions in an image will, in general, identify disjoint regions of the image as sample functions of IEAR(1) processes, i.e. one will not associate the same region of the image with an IEAR(1) process as the other. For example, in regions where the gray level values are low, the ratios of adjacent pixel values fluctuate over a wide range, even when the

gray level values themselves do not change so much. In this case, for the same ϵ crit. 2 will not declare this particular region as the realization of an IEAR(1) process whereas crit. 1 will. Conversely, in regions of large x_i's, the ratios ususally remain close to one (in the absence of edges), though the pixel values themselves may undergo a large variation. Here, for small ϵ, crit. 1 may not be able to identify such a region of the image with an appropriate IEAR(1) process whereas the same is not true for crit. 2. Exactly similar arguments are valid for the IEAR-IC(1) process with p replaced by p_1+p_3 and noting that x_k and x_{k-1} can be interchanged freely there. In chapter 4 performances of these two criteria in image coding are compared.

In the arguments that follow, 'equivalence' of y_k values would imply y_k values being equal in an ϵ-neighbourhood in the sense defined above by the two criteria 1 or 2. The probability p of the ZE condition is assumed to be ≥ 0.5. This would imply that at least every alternate y-value, in a region of the image that can be modelled as a sample function of an IEAR(1) process, is equal to α. However, the algorithm, in order as not to hastily reject a region, which can yield a saving through coding using an IEAR(1) process, allows one instance of two y-values, equal in an ϵ-neighbourhood and differing in two places, to be included in deciding if a particular region is to be coded by the IEAR(1) method. Also, the value of y_k when both x_k and x_{k-1} are zero is taken to be one, and the values when $x_{k-1}=0$ and $x_k \langle\rangle 0$, are assigned the value -1.0 and are not considered while resolving the value of the prediction coeffecient for encoding a particular region of the image.

The method described below uses crit. 2 above in isolating the realizations of different IEAR(1) processes within an image. The straightforward modifications in the method for crit. 1 are described later.

The method proceeds as follows: the y-array, derived from the 1-d bloc obtained from the image, is scanned by a window, $W=\{w_i, i=1 \text{ to } 4\}$, which spans fou successive y-values. W, in the beginning, is stationed at one end of the y-array i.e., $w_i=y_i$, for $i=1$ to 4. The α-value is taken to be y_1. If $y_1=-1.0$ then W shifts one place to the right and the next y-value, viz. y_2, is taken to be the α-value. This is repeated whenever any α-value being considered becomes -1.0. The values of w_2, w_3 and w_4 are next compared with α and W shifts with w_1 at the first w-value which is in an epsilon neighbourhood of α. After this restationing of W, w_2, w_3 and w_4 are again compared with α and W shifted appropriately; simultaneously, a counter keeping the count of the occurance of the ZE conditions is updated. Note that once w_4 is found equal to α, the length of the window is reduced by one, i.e., only the values of w_2 and w_3 are used in determining the equivalence of the W values with α, as explained above. This process is repeated till either the end of the 1-d block is reached or none of w_2, w_3 or w_4 are found to be in an ϵ-nbd. of α. Now, before deciding if this block is to be coded using an IEAR(1) process, it is seen if a sufficient number of ZE conditions, **t**, have occured, over the block length. The sufficiency conditions are examined in the next section, and a large enough t would indicate that this block would achieve better economy if coded using an IEAR(1) process than when it is coded using DPCM. If an enough number of ZE conditions are detected, then the starting and the end points of this block together with the value of the predictor coeffecient are stored. Alternatively, if a sufficient number of ZE conditions are not discerned, W moves back to so that w_1 is is positioned at one place right of the y-value which was taken as α. The value in w_1 is taken as α and the possibility of using this α-value as the predicton coeffecient for coding the following region is then explored, as done previously. If a succession of these α-values is found to yield an unsatisfactory freqency of ZE

conditions, this region cannot be coded using the IEAR(1) process without causing a deterioration in the compression performance, and is coded using the DPCM methodology.

Sometimes, it may be possible to identify a particular region of an image with more than one IEAR(1) process. In a more common situation, the end of the region of validity of one IEAR(1) process may overlap with the beginning of the neighbouring one. By making a judicious choice as to the range over which a particular IEAR(1) process is to be used in coding, better compression than otherwise can be achieved. Hence, after it has been decided to encode a particular region using an IEAR(1) process, as described in the previous paragraph, W is moved back and is stationed with w_1 at the first value in the region not in an ϵ-neighbourhood of α. This value in w_1 is next explored as a prospective value for a new prediction coeffecient, as outlined previously. If a large enough number of ZE conditions is not found, starting with this new seed, W is shifted with w_1 at the next y-value not in an ϵ-nbd. of the α chosen in the region previously. A sufficient number of ZE conditions starting from a seed α, may ultimately either lead to a new IEAR(1) process the region of validity of which is contained entirely within that of a previous IEAR(1) process, or may be partially overlap the region of the previous IEAR(1) process or may be disjoint from it. The entire process, explained above, is repeated till the end of the derived 1-d block is reached and at the end, it yields the starting and the end locations, and also the values of the various associated prediction coeffecients, for the IEAR(1) processes to be used in coding the 1-d block. The procedure to sort out these overlapping regions before coding is outlined in the sec. 3.5.

The arguments for the use of crit. 1 instead of crit. 2 follow in an analogous fashion. Here the window W scans not the y-array but the 1-d block

derived from the original image. An α-value is first calculated as the ratio of two adjacent x-values. After this, the potential of this α as a predicton coeffecient in the region is ascertained, as before. The products of w_1, w_2 and w_3 with α are next formed and W is shifted with w_1 at the position of the first w_{k+1}, if the product of α and w_k lies in an ϵ-nbd. of w_{k+1}, and simultaneously a counter updated. The process is repeated till either the end of the 1-d bock is reached or none of the three products mentioned satisfy the crit. 1. In the actual implementation two windows W_1 and W_2 are used, W_2 moving over x-array as before, and W_1 moving over the y-array and furnishing the α-values to W_2 for determining the occurance of a ZE condition through the use of criterion (2). The two windows scan identical locations in the y and x arrays, respectively and each undergoes the same shifts as the other. The sufficiency conditions for this case remain the same as for crit. 2. At the end of the 1-d block the procedure yields the starting and the end locations of the IEAR/IEAR-IC regions, in general overlapping, and these must be sorted out before the actual prediction/coding process.

The above procedures make the algorithm one of a multipass nature, since it repeatedly scans a given length of the derived 1-d block. The scheme also requires a buffer of the length of the 1-d block, and the maximum admissible length of a potential IEAR/IEAR-IC region is also limited by this length. An example will help to clarify the above procedure:

Example: Consider the case when we are trying to isolate the sample functions of an IEAR(1) process. Let us suppose that t, the number of occurances of a ZE condition, must be at least 3, before a region can be identified as the realization of any IEAR(1) process. For the purpose of this example take ϵ=0.

The sequence y_k has been obtained as:

y_k	0.7	0.9	0.2	0.9	1.0	1.0	0.9	1.0	0.5	0.9	0.3
position	1	2	3	4	5	6	7	8	9	10	11

The procedure sequences as follows:

$t=1$;

STEP 1: w_1 at 1; w_2, w_3, w_4 <> w_1; shift w_1 to 2.

STEP 2: w_1 at 2; $w_3=w_1$; shift w_1 to 4; $t=2$.

STEP 3: w_1 at 4; $w_4=w_1$; shift w_1 to 7; $t=3$.

STEP 4: w_1 at 7; $w_4=w_1$; $w_4=w_1$ already occured in STEP 3; shift w_1 to 3; Extent of the identified IEAR(1) region is from position 2 to position 7; reset t to 1.

STEP 5: w_1 at 3; w_2, w_3, w_4 <> w_1; shift w_1 to 5 (because the y_k value at position 4 already taken in the previous IEAR(1) process).

STEP 6: w_1 at 5; $w_2=w_1$; shift w_1 to 6; $t=2$.

STEP 7: w_1 at 6; $w_3=w_1$; shift w_1 to 8; $t=3$.

STEP 8: w_1 at 8; w_2, w_3, w_4 <> w_1; STOP; Extent of the identified IEAR(1) region is from position 5 to position 8.

Thus, the algorithm in the example results in overlapping IEAR(1) regions, which must be sorted out before coding can be done.

3.4 The criteria for the sufficiency of t

The criterion to determine if a sufficient number of ZE conditions have occured ensures that coding a particular region through an intervening IEAR(1) or an IEAR-IC(1) process would yield a better compression than DPCM. Say that the length of the IEAR(1) region under consideration is **l**, i.e. W has scanned l pixels

before either the end of the 1-d block is encountered, or none of w_2, w_3 or w_4 is found equal to α. Over this region say **t** probable ZE conditions have been discerned. It would take 31 bits to code this region using 3 bit/pel DPCM. If the IEAR(1) process is used to code this region the information would take

$$t+4(l-t)+3+4+1 \text{ bits.}$$

This number is arrived at by considering the coding protocol and the overheads as done in sec.3.1. 3 bits are used to indicate the switch from DPCM to the IEAR(1) coding method, 4 bits to indicate the reverse switch, and 1 bit is used to indicate to the receiver if the region following the IEAR(1) region is coded using an IEAR(1) scheme or a DPCM scheme. If the IEAR(1) method is to secure an economy over DPCM then we must have

$$t+4(l-t)+3+4+1 \leq 31$$

or, $$t > \frac{l}{3}+2$$

However, if the IEAR(1) region is situated at the end of the 1-d block the last 5 bits required to indicate the switch from the IEAR(1) to the DPCM will not be needed in signalling, and the above inequality, in this case, becomes

$$t \geq \frac{l}{3}+1$$

In the case of the IEAR-IC(1) process,the signalling protocol would demand the use of

$$2t+4(l-t)+3+4+1 \text{ bits}$$

to code a length l with t ZE conditions over it. To achieve economy over DPCM one must have

$$t > \frac{l}{2}+4$$

when the IEAR-IC(1) region is not situated at one end of the 1-d block, and

$$t \geq \frac{l}{2}+2$$

when it is. It must be remembered that when the case of α=1.0 arises during the

coding using an IEAR-IC(1) process the coding process reverts to that of the IEAR(1) process and the former two bounds are valid for this case. An idea to the stringency of the above inequalities can be acquired from the tables below which show the bounds used in the implemented algorithm. t_{min} represents the requirement on the minimum number of ZE conditions in a length l for the IEAR(1)/IEAR-IC(1) coding to perform better than DPCM.

l	t_{min}
4	4
5	5
6	5
7	5
9	6
10	6
16	8

$t_{min}=(l \text{ div } 3)+3$

for IEAR(1) process

l	t_{min}
9	9
10	10
11	10
12	11
13	12
14	12
16	13

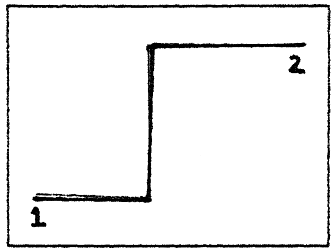
$t_{min}=(l \text{ div } 2)+5$

for IEAR-IC(1) process

It must be mentioned here that all the above inequalities actually give lower bounds for t, since the additional overheads in quantizing and coding the prediction coeffecients, α, are not considered in determining these bounds. The reason for this is explained in sec.3.6.

The problem with the above scheme is that, although it promises the best way of allocating the IEAR/IEAR-IC regions, all the ZE conditions predicted do not occur when the quantization errors are taken into consideration. This can be

illustrated taking the example of an abrupt edge, where the gray level values jump from the minimum to the maximum. The algorithm assigns the entire range 1-2 as the sample function of an IEAR(1) process with α=1.0, and predicts that the NZE condition would occur just once. However, the quantization of the error values would bring about the slope overload error, and consequently it would take a large number of samples before the error would be close to zero. Under these circumstances the number of ZE conditions would be much greater than the predicted value, viz. one.



This loophole in the above scheme paves the way for an alternative method which may perform slightly better than the former scheme. Here, the sufficiency of the ZE conditions is not determined through an inequality as before. Rather, it is determined through the number of ZE conditions exceeding a particular number, **n**, determined experimentally and chosen independently of the length of the region of validity of the IEAR(1)/IEAR-IC(1) process. As n is increased from a small value, the total number of bytes needed to code an image goes through a minimum. This is expected, since when n is small, the number of bits needed to code a particular image by representing it as a succession of IEAR/IEAR-IC regions interspersed with regions to be coded by DPCM, will be larger than if the entire image were to be coded using DPCM alone, because the overall saving occuring from coding the IEAR(1) regions will not be able to offset the extensive overheads caused in coding the regions with a small frequency of ZE conditions. At the other end, if n is kept too large, then again the bit rate increases, for the regions which could have yielded a saving are now left out because the decision of coding a region using the IEAR/IEAR-IC method now requires an overly high frequency of the occurance of ZE conditions. Thus there exists an optimum value of n, n_{opt}, which yields the

best compactions. This minimum is found to be relatively broad so that the values of n adjacent to n_{opt}, i.e. $n_{opt}\pm1$, approximatey, yield almost the same bit rates, for a given SNR. The value of n_{opt} remains nearly constant for various real life imagery. This value of n_{opt} is determined experimentally for each particular configuration of the algorithm, and then this n_{opt} is used to decide whether a suffucient number of ZE conditions have occured over a region.

3.5 Sorting the overlapping IEAR(1)\IEAR-IC(1) regions

The overlapping regions, obtained after the above process is over, must be sorted out. The procedure outlined below chooses the combination of IEAR and DPCM regions which would yield the best possible savings. It disregards the interference caused due to the quantization errors.

Let (a_1,b_1,α_1) and (a_2,b_2,α_2) be the starting point, the end point and the values of the prediction coeffecient respectively for two regions to be coded using different IEAR models. The following three cases arise here:

Case 1: If $a_2>b_1$ then the two regions are disjoint and are coded in turn by their respective α's.

Case 2: If $a_2<b_1$ and $b_2\leq b_1$ then the second region is wholly contained within the first, and the region (a_1,b_1) is coded using α_1 as the predictor coeffecient without any further attention being paid to the region contained inside the first.

Case 3: If $a_2<b_1$ and $b_2>b_1$ then the regions overlap. By a method similar to that above, it is determined if the coding of the regions (a_1,a_2,α_1) and (b_1,b_2,α_2) using their respective IEAR processes secures an economy over DPCM. If both these regions yield a saving then if the saving in (a_1,a_2)>saving in (b_1,b_2), then (a_1,a_2) is coded using α_1, and (a_2,b_2) is coded using α_2; otherwise (a_1,b_1) is coded using α_1 and (b_1,b_2) is coded using α_2. If (a_1,a_2,α_1) yields a saving over DPCM and (b_1,b_2,α_2)

does not then (a_1,a_2) is coded using α_1, and (a_2,b_2) is coded using α_2, and vice-versa. If adequate saving is not achieved in either (a_1,a_2) or in (b_1,b_2) then out of the two alternatives, coding (a_1,b_1) using α_1 and coding (a_2,b_2) using α_2, the one yielding a greater reduction in the number of bytes is taken and the other option discarded.

All the (a_i,b_i,α_i) values for a 1-d block are paired and the procedure outlined above repeated for each pair. At the end it yields the sequence of non-ovarlapping IEAR(1)/IEAR-IC(1) regions which can code the given 1-d block in the most economical way.

3.6 Differential coding of the image using the IEAR(1)/IEAR-IC(1) processes

The non-overlapping regions obtained as a result of the above procedures can be coded as explained previously: the IEAR(1) regions by the scheme in sec. 2.7 and the IEAR-IC(1) regions in sec. 2.10. The intervening regions are coded by straightforward DPCM. The value of the predicition coeffecient in case of DPCM is taken to be 1.0. The reason for this is the fact that horizontal correlations for most of natural imagery are estimated to be around 0.96 [5]. Hence an elegant and not too wasteful approximation is to choose the α for DPCM coding as 1.0.

The estimate for the first pixel in an image, $\hat{x}^R_{0,0}$ is taken to be the maximum value of the PCM quantization level in the image (e.g. for a 6 bit/pel image $\hat{x}^R_{0,0}=63$). For the successive blocks, the estimate for the first sample in the block is taken to be the reconstructed value of the pixel adjacent to it, i.e., when the 1-d blocks are obtained from the image then $\hat{x}^R_{i,0} = x^R_{i-1,0}$; otherwise when the 1-d blocks are obtained from the image on concatenating the rows of the 2-d blocks (4x4 or 8x8), into which the image is sub-divided, the estimate for the

first pixel of each 1-d block is taken to be the reconstructed value of the pixel adjacent to it in one of the neighbouring blocks.

Now we come to the problem of the two quantizations involved in the encoding procedure: the quantization of the prediction coeffecients, α, and the quantization of the error, $\{e_i\}$. The quantization of the α's is taken up first.

In the absence of quantization noise in the prediction errors, it is possible to do away with the quantization of the α's. Let x_1 be the first sample value in a region which is to be coded using the IEAR coding scheme. Let x_0 be the pixel that comes immediately before x_1. If the actual error values could be sent, with no quantization noise, then the predicted value for x_1, $\hat{x}_1$, could be taken as x_0, and the error between x_1 and $\hat{x}_1$ transmitted to the receiver. The receiver would now evalauate α by finding the ratio x_1/x_0. This α-value would now be used by both the transmitter as well as the receiver as a prediction coeffecient in forming further estimates. Hence in this scheme, the estimate for the first sample in an IEAR region is formed taking α as 1.0. Here, the α-value valid for the entire IEAR region can be evaluated from two initial values, and the brunt of transmitting these α's can be avoided.

If the quantizing noise is present, then the receiver forming the estimate of α from the ratio x^R_1/x^R_0 does not necessarily yield a value in the neighbourhood of the real α. An error in the value of α reflects as a sharp fall in the frequency of the ZE conditions, and the accompanying degradation in the compression performance. In such a case, the value of α must be quantized, coded and transmitted to the receiver. For the IEAR-IC(1) process the value of α lies in the range (0,1]. Here a first approach is to quantize this entire range. In the implementations used, a 5-bit quantizer is employed for this purpose. Improvements

in this quantization scheme stem from the realization that the most common α values are those that lie close to 1.0. Here, the values of α lying outside a small range around 1.0 are not considered while assigning the values of the prediction coeffecients for various IEAR regions, by the method in sec.3.3, while the α values lying inside this range are quantized and coded. If the coding is to be done through the IEAR-IC(1) process, $|\alpha| \leq 1.0$ and the quantization intervals for α are as shown below

80 83	84 85	86 88	89 91	92 94	95 96	97 98	99 100	$\times 10^{-2}$
81	83	87	90	93	96	98	100	$\times 10^{-2}$

In IEAR(1) coding, the α values can be greater than 1.0 and the following quantization scheme is used

86 88	89 91	92 94	95 96	97 98	99 101	102 104	105 106	$\times 10^{-2}$
87	90	93	96	98	100	103	105	$\times 10^{-2}$

In the two tabulations above, the values lying in the first row in the range indicated are quantized to the values in the the second row.

Now the details of the quantization of the error are discussed. It has been shown experimentally that images with widely varying distributions of pixel values have a prediction error which is Laplacian. Hence, the prediction error is assumed to have a Laplacian density, and consequently a non-uniform quantizer, such as the Lloyd-Max quantizer, would perform better than a uniform quantizer. In the experiments on the natural images done, a Lloyd-Max quantizer is used for the purpose of error quantization. The decision and the reconstruction levels for a 3

bit\pel quantizer for a signal having a unit variance Laplacian density are shown in Table 3.6.1 below [5].

d_i	r_i
$-\infty$	-3.0867
-2.3796	-1.6725
-1.2527	-0.8330
-0.5332	-0.2334
0.0000	0.2334
0.5332	0.8330
1.2527	1.6725
2.3796	3.0863
$+\infty$	

Decision and reconstruction levels for a 8 level non-uniform quantizer, assuming the signal, to be quantized, to have a unit variance Laplacian density.

Table 3.6.1

Before this quantizer can be used, the values in the table 3.6.1 must be multiplied by the standard devations of the prediction error. The estimates of the standard deviation of the prediction error are formed using the relation

$$\hat{\sigma}=\sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}e^{2}_{i}\right)}$$

These estimates for various test images are shown in Tables 3.6.2 and 3.6.3.

Image Name	$\hat{\sigma}_{DPCM}$	$\hat{\sigma}_{IEAR-IC}$	$\hat{\sigma}_{IEAR}$
1. Lisa	10.03	10.12	10.03
2. Lincoln	7.85	7.92	7.85
3. Stalib	11.08	10.12	11.08

Estimates for prediction error standard deviations for 6 bits/pel images

Table 3.6.2

Image Name	$\hat{\sigma}_{DPCM}$	$\hat{\sigma}_{IEAR}$
1. Cafe	24.03	25.27
2. Plath	10.88	13.38
3. Eliot	13.15	14.98
4. Sat1	11.37	13.28
5. Sat2	13.06	15.90
6. CafeHQ	33.27	33.76
7. PlathHQ	21.86	21.89
8. EliotHQ	21.20	21.91

Estimates for prediction error standard deviations for 6 bits/pel images

Table 3.6.3

A final modification in the quantizer design procedure is required. The reason for this is that one code word is required to signal to the receiver the switch from the DPCM scheme to the IEAR/IEAR-IC scheme or the reverse transition. As such, only 7 out of a possible 8 levels are available for the quantization of the error. Hence the levels in Table 3.6.1 must be modified, by solving the design equations [5]. In this work this modification is made intuitively. First, the range over which the levels of the 8-level quantizer will vary is determined by multiplying the values in Table 3.6.1 by the maximum and minimum standard deviation values in tables 3.6.2 or 3.6.3. This is necessary because it is assumed that the same quantizer is being used for all the images. Bearing in mind this variation in the decision or reconstruction levels, and also the non-uniform nature of the quantizer the number of levels may be reduced by one in a heuristic manner. These quantizers for the quantizing the prediction error to 3 bits for 6 bits/pel and 8 bits/pel images are shown below:

-63 -26	-25 -10	-9 -3	-2 2	3 9	10 25	26 63
-34	-18	-6	0	6	18	34

7-level prediction error quantizer for 6 bit/pel images

-255 -50	-49 -22	-21 -6	-5 5	6 21	22 49	50 255
-64	-35	-12	0	12	35	64

7 level prediction error quantizer for 8 bit/pel images

Once the quantization scheme has been finalized, the actual details of coding can be discussed. The procedures desribed in sections 3.3-3.5 yield the starting points, the end points and the α values for the IEAR(1)/IEAR-IC(1) regions

by means of 4 bits and the following bit indicates if the following region is to be coded using DPCM or the IEAR process. The procedure is carried on till the end of the 1-d block ,and is repeated for each 1-d block.

Note that the scheme for the non-adaptive coding of images using the IEAR(1)/IEAR-IC(1) process follows in a natural way. A suitable value of α is first chosen and each 1-d block is coded using this value of α. Since the entire image is assumed to be the sample function of IEAR/IEAR-IC process with this α, no regions are coded using DPCM. Although this scheme has the disadvantage that a much higher number of NZE conditions, than the adaptive scheme, have to be coded using a longer (4 bit) word, the overheads required to indicate the switch between the IEAR(1)/IEAR-IC(1) coding strategy and the DPCM are eliminated in this case. An additonal attraction of the the non-adaptive method is its minimal implementation complexity.

This brings us to the end of the discussion on the sotware implementation of the algorithm for image coding based on the IEAR(1)/IEAR-IC(1) processes. In the next chapter, we shall compare the compression performance of the various configurations of the algorithm described here.

Chapter 4

Results and Conclusions

In this chapter, we compare the performance, in the coding of various real
: imagery, of the various algorithms catalogued in Chapter 3. This evaluation is
ieved by analyzing the inter-relationships between the implementation
plexity, the compressions achieved and the distortion caused in the experiments
the coding and the reconstruction of suitably chosen test images.

The main result that we shall attempt to arrive at, at the end of this
pter, is that all of the configurations of the algorithm for image coding, using
IEAR(1)/IEAR-IC(1) processes, result in a coding performance which is always
se than a scheme of variable length coding of the quantized prediction error
ues obtained from a closely related DPCM scheme. The route to this result,
ever, would yield valuable spinoffs in the form of insight into the right choice
parameter values, such as the range over which a prediction error value may be
nalled as equal to zero, without any significant deterioration in the image
lity.

The test images assume the PCM coded amplitude resolutions of 6 bits/pel
8 bits/pel, respectively. The two 6 bits/pel images are of the head and

shoulder type, and are named LISA and LINCOLN. The 8 bits/pel images include an image of an outdoor scene, named CAFE, a head and shoulder image, named PLATH, and a satellite image, named SAT. The coding of the histogram equalized versions of the 8 bit/pel images has also been investigated.

Although appropriate remarks have been included regarding the visual appearance of an image, coded and reconstructed using either DPCM or a scheme based on the IEAR(1)/IEAR-IC(1) process, to establish an objective comparison between the algorithms their performance must be viewed quantitatively. The coding schemes based on the IEAR(1)/IEAR-IC(1) process suffer from the disadvantage that neither the number of bytes, nor the signal to quantization noise ratio (hereafter referred to as SNR) is known until the actual coding has been completed; in other words, neither of these two quantities can be fixed a priori. Therefore, both the number of bytes used in coding the image, and the SNR's obtained must be examined together, before an understanding of the relative capabilities of the various algorithms is acquired. Here, we first proceed to identify the best configurations among the adaptive schemes, explained in Chapter 3, and then compare their performance with the non-adaptive method.

4.1 Performance of adaptive algorithms

The results for the adaptive coding of the images PLATH and CAFE, using a 3 bit quantization code-word for the prediction coeffecient α, are shown in the Tables 4.1.1-4.1.4. Because the sizes of these images is larger than those for 6 bits/pel images, this helps to amplify the differences between the various algorithms. In these tables crit. 1, crit. 2, crit. 1' and crit. 2' refer to the criteria used in isolating the IEAR(1)/IEAR-IC(1) regions (sec. 3.3) and the criteria used in signalling the ZE conditions (sec. 3.6), respectively. The condition $t \geq n(l)$ means

PLATH

	Row-wise scanning															
	coding using crit. 1′								coding using crit. 2′							
	assignment using crit. 1				assignment using crit. 2				assignment using crit. 1				assignment using crit. 2			
	IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)	
	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$
		$n_{opt}=7$		$n_{opt}=7$		$n_{opt}=6$		$n_{opt}=9$		$n_{opt}=6$		$n_{opt}=7$		$n_{opt}=9$		$n_{opt}=12$
$\varepsilon-\varepsilon'$	5-8	5-8	5-8	5-8	2-8	2-8	2-8	2-8	5-7	5-7	5-7	5-7	2-4	2-3	2-3	2-3
bytes	4262	4238	4190	4244	4526	4519	4146	4217	4089	4092	4044	4115	4703	4815	4688	4641
SNR	33.39	33.35	33.18	33.29	33.37	33.36	33.09	33.26	30.69	30.59	29.46	29.06	33.34	33.66	32.39	32.55
$\varepsilon-\varepsilon'$	8-15	8-10	8-15	8-15	2-15	2-15	2-15	2-15	5-15	--	5-15	--	2-7	2-7	--	2-7
bytes	3777	4074	3764	3803	4304	4326	3804	3879	3807	--	3821	--	4390	4506	--	3974
SNR	28.76	32.17	29.09	29.17	30.20	30.42	29.15	29.11	25.91	--	24.91	--	30.41	30.72	--	28.55
$\varepsilon-\varepsilon'$	15-15	15-15	15-15	--	4-15	4-15	4-15	4-15	8-10	8-10	8-10	8-10	4-7	4-7	4-7	4-7
bytes	3861	3814	3893	--	3871	3843	3802	3827	3746	3739	3794	3836	4006	3971	4117	4171
SNR	28.45	28.36	28.73	--	29.10	29.12	28.73	28.75	28.26	27.89	25.72	25.48	30.74	30.56	24.80	24.71
$\varepsilon-\varepsilon'$	8-8				4-8								4-15			
bytes	4358				4413								3625			
SNR	28.42				33.57								25.51			

DPCM coding results			
7-level quantizer		8-level quantizer	
no. of bytes	6144	no.of bytes	6144
SNR	34.30 dB	SNR	35.75 dB

Comparison of experimental results for the image PLATH

Table 4.1.1

PLATH

	block-wise scanning															
	coding using crit. 1′								coding using crit. 2′							
	assignment using crit. 1				assignment using crit. 2				assignment using crit. 1				assignment using crit. 2			
	IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)	
	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$
		$n_{opt}=6$		$n_{opt}=6$		$n_{opt}=8$		$n_{opt}=7$		$n_{opt}=6$		$n_{opt}=6$		$n_{opt}=6$		$n_{opt}=9$
ϵ-ϵ'	5-8	5-8	5-8	5-8	2-8	2-8	2-8	2-8	5-7	5-7	5-7	5-7	2-3	2-3	2-3	--
bytes	4586	4513	4485	4497	4763	4867	4509	4532	4373	4358	4278	4377	5142	5023	5051	--
SNR	31.49	31.55	31.69	31.61	31.68	31.82	31.61	31.66	30.73	30.90	30.62	30.36	31.79	31.86	31.04	--
ϵ-ϵ'	8-15	8-15	8-15	8-15	2-15	2-15	2-15	2-15	8-10	8-10	8-10	8-10	2-7	2-7	2-7	2-7
bytes	4055	4046	4084	4058	4467	4776	4151	4203	4039	4032	4020	4086	4563	4634	4278	4410
SNR	29.43	29.62	29.65	29.76	30.24	31.27	29.98	30.09	28.97	29.23	28.09	28.00	30.73	31.18	29.56	30.25
ϵ-ϵ'	15-15	15-15	15-15	--	4-15	4-15	4-15	4-15	8-15	8-15	8-15	8-15	4-7	4-7	4-7	4-7
bytes	4025	4001	3986	--	4095	4140	4048	4044	3846	3872	3932	4001	4264	4231	4266	4281
SNR	29.09	29.16	29.30	--	29.57	30.12	29.32	29.40	27.34	27.80	26.50	26.26	30.40	30.51	26.91	27.21
ϵ-ϵ'	8-8				4-30		4-30						4-15		4-15	
bytes	4501				3839		3905						3867		3987	
SNR	31.57				26.88		26.11						27.46		24.00	

DPCM coding results			
7-level quantizer		8-level quantizer	
no. of bytes	6144	no.of bytes	6144
SNR	31.94 dB	SNR	32.62 dB

Comparison of experimental results for the image PLATH

Table 4.1.2

CAFE

	Row-wise scanning															
	coding using crit. 1'								coding using crit. 2'							
	assignment using crit. 1				assignment using crit. 2				assignment using crit. 1				assignment using crit. 2			
	IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)	
	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$
		$n_{opt}=7$		$n_{opt}=7$		$n_{opt}=6$		$n_{opt}=9$		$n_{opt}=6$		$n_{opt}=7$		$n_{opt}=9$		$n_{opt}=12$
ε-ε'	5-8	5-8	5-8	5-8	2-8	2-8	2-8	2-8	5-7	5-7	5-7	5-7	2-4	2-3	2-3	2-3
bytes	5678	5642	5634	5652	5756	5735	5623	5648	5553	5539	5515	5571	5802	5834	5871	5824
SNR	28.92	28.95	28.88	28.92	28.91	28.93	28.85	28.87	28.30	28.39	28.10	28.08	28.86	28.96	28.85	28.87
ε-ε'	8-15	8-10	8-15	8-15	2-15	2-15	2-15	2-15	5-15	--	--	--	2-7	2-7	--	2-7
bytes	5350	5543	5306	5398	5626	5629	5389	5475	5374	--	--	--	5648	5726	--	5601
SNR	27.34	28.54	27.46	27.44	28.24	28.33	27.73	27.95	26.80	--	--	--	28.35	28.54	--	28.06
ε-ε'	15-15	15-15	15-15	15-15	4-15	4-15	4-15	4-15	8-10	8-10	8-10	8-10	4-7	4-7	4-7	4-7
bytes	5260	5170	5188	5288	5406	5377	5287	5359	5333	5303	5268	5393	5474	5474	5456	5538
SNR	26.77	26.84	26.89	26.85	27.57	27.63	27.27	27.20	26.99	27.19	26.36	25.95	27.93	28.18	25.50	25.30
ε-ε'	8-8				4-8				8-15				4-15		4-15	
bytes	5794				5777				5134				5186		5104	
SNR	28.83				28.87				25.59				25.59		22.38	

DPCM coding results			
7-level quantizer		8-level quantizer	
no. of bytes	6144	no.of bytes	6144
SNR	29.04 dB	SNR	29.57 dB

Comparison of experimental results for the image CAFE

Table 4.1.3

CAFE

	block-wise scanning															
	coding using crit. 1′								coding using crit. 2′							
	assignment using crit. 1				assignment using crit. 2				assignment using crit. 1				assignment using crit. 2			
	IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)	
	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$
	$n_{opt}=6$		$n_{opt}=6$		$n_{opt}=8$		$n_{opt}=7$		$n_{opt}=6$		$n_{opt}=6$		$n_{opt}=6$		$n_{opt}=9$	
ϵ-ϵ'	5-8	5-8	5-8	5-8	2-8	2-8	2-8	2-8	5-7	5-7	5-7	5-7	2-3	2-3	2-3	2-3
bytes	5764	5584	5608	5597	5790	5736	5594	5619	5575	5486	5464	5532	6018	5791	5857	5801
SNR	25.59	25.63	25.70	25.59	25.63	25.69	25.62	25.62	25.38	25.46	25.32	25.29	25.67	25.67	25.52	25.62
ϵ-ϵ'	8-15	8-15	8-15	8-15	2-15	2-15	2-15	2-15	8-10	8-10	8-10	8-10	2-7	2-7	2-7	2-7
bytes	5347	5267	5277	5315	5564	5677	5372	5449	5304	5259	5262	5327	5620	5602	5452	5591
SNR	25.01	25.10	25.13	25.10	25.32	25.59	25.24	25.36	24.78	25.05	24.48	24.39	25.42	25.51	25.11	25.42
ϵ-ϵ'	15-15	15-15	--	--	4-15	4-15	4-15	4-15	8-15	--	8-15	--	4-7	4-7	4-7	4-7
bytes	5302	5187	--	--	5366	5369	5291	5289	5120	--	5137	--	5481	5389	5460	5461
SNR	24.67	24.82	--	--	25.63	25.35	24.87	24.92	23.91	--	23.62	--	25.20	25.37	23.59	24.11
ϵ-ϵ'	8-8				4-30	4-25	4-30						4-15		4-15	
bytes	5767				5087	5278	5061						5132		5145	
SNR	25.56				23.75	24.97	23.16						24.00		21.62	

DPCM coding results			
7-level quantizer		8-level quantizer	
no. of bytes	6144	no.of bytes	6144
SNR	25.71 dB	SNR	25.94 dB

Comparison of experimental results for the image CAFE

Table 4.1.4

that one of the inequalities mentioned in sec. 3.4 are used in determining if a sufficient number of ZE conditions has occured so that coding using the IEAR/IEAR-IC results in a saving over DPCM. The condition $t \geq n_{opt}$ implies the alternative case where a region is to be coded using an IEAR/IEAR-IC process if the number of ZE conditions in the region exceed an experimentally determined number n_{opt}. The number of bytes required to code the image and the signal to quantization noise ratios are indicated for each case for the values of ϵ and ϵ' shown. Although the comparisons between the various versions of the algorithms are difficult to make, for no version is distinctly the best one, these tables enable us to make the following general observations about the relative performance of the variants of the coding process:

The 7-level quantizer quantizes a certain range of values of the prediction errors in the nbd. of zero as zero. (For 8 bit/pel PCM coded images, it was shown in sec. 3.6 that the range [-5,5] is quantized as zero). As the interval over which a prediction error, if it lies in this interval, is signalled as a ZE condition (i.e. the value of ϵ', if the coding is done using crit. 1') is increased a fall in SNR, corresponding to a considerable deterioration in image quality, is detected. This is easily seen in Table 4.1.1, where the increase of ϵ' from 8 to 15, when the ZE conditions are signalled using crit. 1', causes the SNR's to fall from about 33.3 dB to less than 30 dB, without an accompanying increase in the compression ratios. Again, when the regions are isolated in accordance with crit. 1, and ZE conditions are signalled using crit. 2', an increase of ϵ' from 3 to 7 causes a similar drop in SNR. The quantizer quantizes the range [6,21] as 12 (and the range [-21,-6] as -12) and the more ϵ' erodes into this range, the greater is the degradation in image quality. In Table 4.1.1, as long as $\epsilon' \lesssim 10$ (for the case when the ZE conditions are siganalled using 1' as the criterion), this decrease of SNR's is not observed. In the

with the use of the condition $t \geq n(1)$, and the IEAR(1) process with, the use of the inequality $t \geq n_{opt}$, in cases where crit. 1′ is used in signalling the ZE conditions. From the above discussion we conclude that the use of crit. 2 in identifying the image regions as the sample functions of IEAR(1) processes, with either crit. 1′ or crit. 2′ in signalling the ZE conditions, and the use of crit. 2 in isolating the IEAR-IC(1) regions, with the use of crit. 1′, yield a compression performance which is roughly equivalent. If crit. 1 is used, the IEAR(1) process and the IEAR-IC(1) process, with the use of crit. 1′ in indicating the ZE conditions, yield rougly similar compression ratios which are better than those obtained through the use of crit. 2′.

For 8 bits/pel images, when crit. 1′ is used in coding, the schemes using either the IEAR(1) or the IEAR-IC(1) processes, yield almost the same performance regardless of whether the regions have been isolated using crit. 1 or crit. 2. However, the IEAR(1) and the IEAR-IC(1) processes, with the use of the criteria 1 and 1′, for assigning the regions and coding the ZE conditions, respectively, yield slightly better results than the use of crit. 2 and crit. 2′, for the coding scheme using the IEAR(1) process. For 6 bit/pel images, however, the schemes in which the regions have been isolated using crit. 1 perform better than those in which crit. 2 has been used, for both the IEAR(1) as well as the IEAR-IC(1) processes.

The histogram equalization of an image is achieved by replacing the gray level value L_k by the value L'_k, for all pels with the gray level L_k, with L'_k given by

$$L'_k = \sum_{i=1}^{k} \frac{n_i}{n} L_k$$

where n_i is the number of pels at L_i and n is the total number of pels in the image.

Although the relative performance of these algorithms, for histogram equalized

images, follows the pattern outlined above, some interesting trends show up. Firstly, if crit. 2 is used in identifying the regions, the compression ratios achieved are significantly poorer than those achieved in the case of non-histogram equalized images, also the compressions do not improve significanly as the value of ϵ' is increased. This shows that the recurrence of equal adjacent-pixel-ratio values, for these images, is infrequent. The use of crit. 2, however, suffers from no such limitation, and it is possible in this case to obtain higher compression ratios, with larger values for ϵ', without any significant sacrifice of SNR's. However, the ratios obtained for these images are still worse than those obtained for non-histogram equalized ones.

Let us consider the case when the condition $t \geq n_{opt}$ is being used to determine if a sufficient number of ZE conditions have occured, before deciding to code a region using the IEAR/IEAR-IC process. It is interesting to consider the compression performances when the bound for t is in a nbd. of n_{opt}, i.e. the condition being used is $t \geq n$, n being in a nbd. of n_{opt}. Representative values are shown in Table 4.1.5. The value of n_{opt} makes its appearance with a decrese in the no. of bytes used in coding the image, and a simultaneous, but slight increase in the SNR, as n is increased from n_{opt}-1 to n_{opt}. However, compressions in the nbd. of n_{opt} are close to what they are at n_{opt} (differing by s little as about 0.1 percent), so that the minimum of the compression ratios is a flat minimum. A consequence of this result is that the value of n for which the compression is mininimum has been observed to remain approximately constant (remaining in the range $n_{opt} \pm 1$) for various images and also for different values of the parameters ϵ and ϵ'.

The tables 4.1.1 to 4.1.4 give the results for 3-bit quantization of the predictor coeffecient α. The majority of α values, for most real-life, images lie in

between 0.9 and 1.06, for the IEAR(1) process, and between 0.9 and 1 for the IEAR-IC(1) process, and a 5-bit quantization of the entire range of possible α values, i.e. [0,1], for the IEAR-IC(1) process, is therefore wasteful. An example is shown in Table 4.1.6 and a comparison with the values in Table 4.1.1 shows that the performance for the coding procedure using 5-bit quantization of α is found to be consistently worse than that for the IEAR-IC(1) scheme which quantizes the α-value to 3 bits.

A few final remarks will help to clarify the above discussion and summarize the main results. We have seen that the use of crit. 1' in indicating the occurance of ZE conditions performs better than the use of crit. 2', paticularly when the regions have been isolated in accordance with crit. 1. The use of crit. 1' is also to be preferred because here the value of ϵ' (and also ϵ) can be identified directly with the quantizer characteristic, and the possibility of inordinately low SNR's can be minimized. For example, for the 7-level quantizer characteristic for 8 bits/pel images, given in sec. 3.6, the choice of $\epsilon' \lesssim 10$ causes no substantial depreciation in the SNR values, but $\epsilon'=15$ causes a fall of SNR by about 4 dB for the image PLATH. Similarly, for the quantizer for 6 bits/pel images $\epsilon' \lesssim 4$ yields SNR's comparable with 3 bit DPCM, whereas the choice of $\epsilon' \gtrsim 7$ causes a drop in SNR values by 2-3 dB. However, this deterioration in SNR's is depends on the image being coded, and also on the specifics of the algorithm being used and the scanning methodology employed. The drops in SNR observed for PLATH in the row-wise scanning scheme are not observed in the case when the image is sub-divided into smaller blocks.

In Table 4.1.7 are given the percentage compresion relative to 3-bit (7-level) DPCM and the falls in SNR's relative to the latter, for different images, using crit. 1 in isolating the image regions as sample functions of IEAR(1)/IEAR-

CAFE								
IEAR(1) assignment using crit. 2 coding using crit. 2' $\epsilon=4$, $\epsilon'=7$ $n_{opt}=6$			IEAR(1) assignment using crit. 1 coding using crit. 1' $\epsilon=8$, $\epsilon'=15$ $n_{opt}=6$			IEAR-IC(1) assignment using crit. 1 coding using crit. 1' $\epsilon=8$, $\epsilon'=15$ $n_{opt}=6$		
n	no. of bytes	SNR, dB	n	no. of bytes	SNR, dB	n	no. of bytes	SNR, dB
5	5394	25.34	5	5290	25.08	5	5346	25.10
6	5389	25.37	6	5267	25.10	6	5315	25.10
7	5407	25.39	7	5292	25.23	7	5330	25.10
8	5428	25.44	8	5315	25.25	9	5398	25.37

Compresion performance with the bound on t, n, in the nbd. of n_{opt}

Table 4.1.5

FLATH				
Row-wise scanning				
coding using crit. 1'				
	assignment using crit. 1		assignment using crit. 2	
	$t \geq n(1)$	$t \geq n_{opt}$	$t \geq n(1)$	$t \geq n_{opt}$
		--		$n_{opt}=9$
$\varepsilon-\varepsilon'$	5-8	--	2-8	2-8
bytes	4395	--	4554	4619
SNR	33.21	--	33.42	33.55
$\varepsilon-\varepsilon'$	5-15	--	4-15	4-15
bytes	4162	--	4130	4142
SNR	29.86	--	29.08	28.98
$\varepsilon-\varepsilon'$	6-15			
bytes	4062			
SNR	29.22			

The performance of the coding scheme based

on the IEAR-IC(1) process using 5-bit quantization of α

Table 4.1.6

Row-wise scanning

CAFE				PLATH				CAFEHQ			
IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)	
a	b	a	b	a	b	a	b	a	b	a	b
8.2	0.09	8.3	0.16	31.0	0.95	31.8	1.12	3.3	0.03	2.8	0.0[illegible]
9.8	0.50	13.6	1.58	33.7	2.13	38.7	5.21	7.9	0.54	4.0	0.3[illegible]
15.9	2.20	15.6	2.15	37.9	5.94	36.6	5.57	---	----	---	----

PLATHHQ				LISA				LINCOLN			
IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)		IEAR(1)		IEAR-IC(1)	
a	b	a	b	a	b	a	b	a	b	a	b
21.4	0.18	18.5	0.12	2.9	0.02	4.2	-0.15	10.9	-0.15	10.3	-0.15
25.3	0.66	24.1	1.96	11.7	0.28	11.9	-0.20	25.1	0.49	23.2	0.5[illegible]
---	---	---	---	16.9	0.32	16.2	0.32	25.1	0.49	23.2	0.5[illegible]

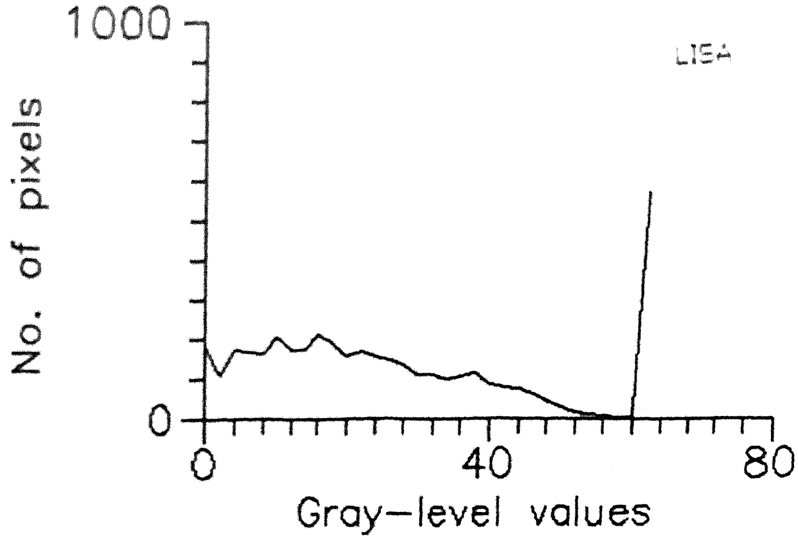
Notation:

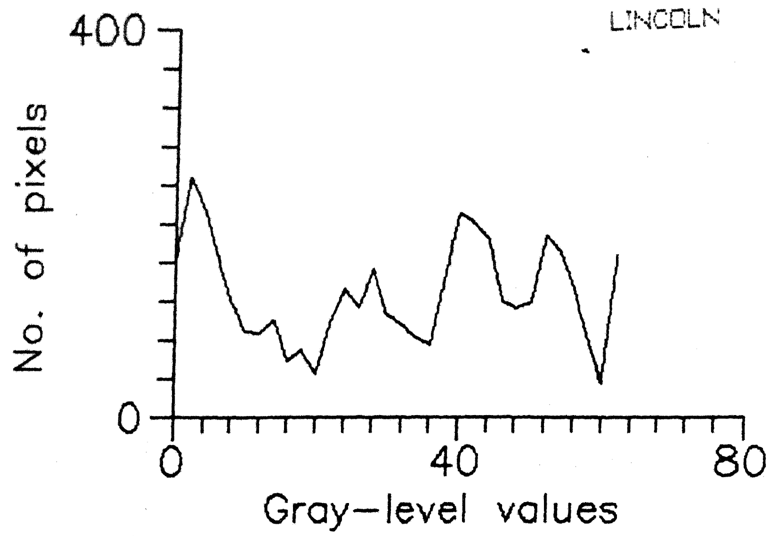
a:Percentage improvement in compression over DPCM

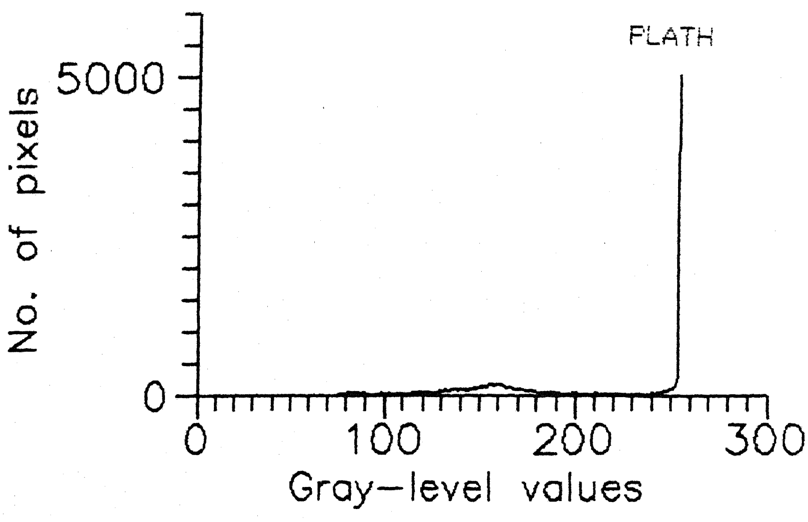
b:Decrease in SNR over DPCM in dB

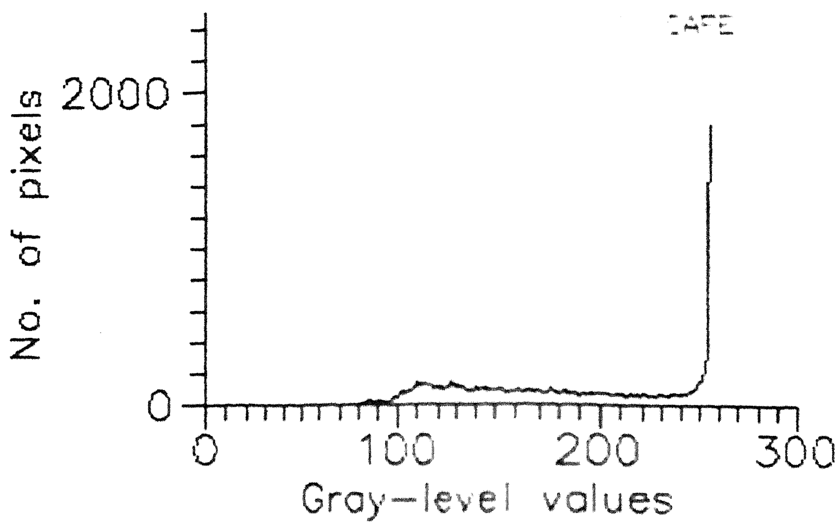
Performance of the coding processes using the IEAR(1) proocess, using $t \geq n_{opt}$, and the IEAR-IC(1) process, using $t \geq n(1)$, relative to DPCM.

Table 4.1.7

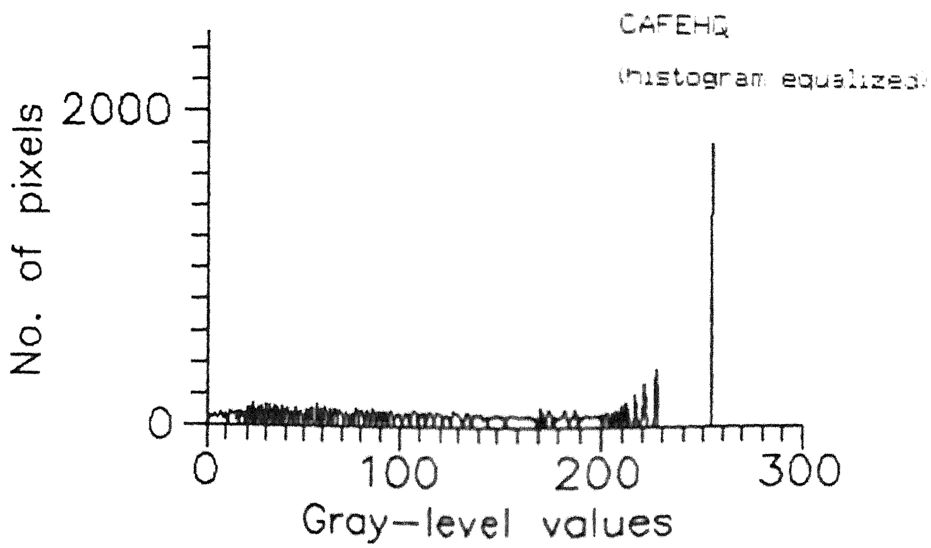

LISA
1000
0
No. of pixels
0
40
80
Gray-level values


LINCOLN
400
0
No. of pixels
0
40
80
Gray-level values


PLATH
5000
0
No. of pixels
0
100
200
300
Gray-level values



CAFE
2000
0
No. of pixels
0
100
200
300
Gray-level values



CAFEHQ
(histogram equalized)
2000
0
No. of pixels
0
100
200
300
Gray-level values

IC(1) processes, and using crit. 1' in indicating the ZE conditions. These values are close to the best values which can be obtained using the implementation of the adaptive schemes considered here. The blockwise methods give similar results. The table shows that compressions from about 10 to 25 percent are possible (for non-histogram equalized images) with almost no degradation in the picture quality. The higher compressions in the case of PLATH are expected because it contains long runs of pixels, all of the value 255. For a more complicated image, like CAFE modest compressions are achieved. (See the histograms of these images on page **73**. The images themselves are displayed at the end of this chapter). The fall in SNR corresponds with a deterioration in image quality, which shows up as either horizontal streaks of equal intensity, in the case of row-wise scanning, or as blobs of equal pixel values, in the case of block-wise scanning.

4.2 Performance of non-adaptive methods

In the non-adaptive schemes a value of α is decided a priori, and then the entire image is coded using this α as the value of the prediction coeffecient, for a coding scheme based on either the IEAR(1) or the IEAR-IC(1) processes, using either crit. 1' or crit. 2' to signal the ZE conditions. For example, for the IEAR(1) process based coding method using crit. 1' to indicate the ZE conditons, whenever the magnitude of the prediction error is $\leq \epsilon'$, a zero is transmitted; if, on the other hand, the magnitude exceeds ϵ', a one, followed by a 3 bit codeword for the quantized error is transmitted. Note that the switch between the DPCM and the IEAR/IEAR-IC coding strategies does not occur here.

To determine if there is a single coding scheme which consistently gives the best compression performance, we follow the following procedure: Since we know that the most common α-values lie in a nbd. of 1.0, for a given image, from a series

PLATH

	row-wise scanning											
	coding using crit. 1′						coding using crit. 2′					
	IEAR(1)			IEAR-IC(1)			IEAR(1)			IEAR-IC(1)		
	$\epsilon'=5$	$\epsilon'=8$	$\epsilon'=15$	$\epsilon'=5$	$\epsilon'=8$	$\epsilon'=15$	$\epsilon'=3$	$\epsilon'=7$	$\epsilon'=15$	$\epsilon'=3$	$\epsilon'=7$	$\epsilon'=15$
α	--	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
bytes	--	6065	4838	5965	5257	4392	--	4752	3504	6433	5103	4707
SNR	--	33.33	27.38	33.82	33.58	29.48	--	27.60	22.22	27.64	21.24	17.83
α	0.99	0.99	0.99	0.98	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.98
bytes	5068	3923	3187	4932	4742	4527	4676	3367	2627	5451	4928	4530
SNR	34.42	32.13	27.30	35.20	33.34	29.08	32.60	27.38	22.35	27.60	24.65	19.58
α	1.00	1.00	1.00	0.99	0.99	0.99	1.00	1.00	1.00	0.99	0.99	0.99
bytes	4154	3497	2924	5183	4896	4593	4200	3132	2561	6372	4773	4436
SNR	34.30	32.77	27.94	34.94	33.18	28.89	33.35	29.19	22.05	28.06	26.10	20.03
α	1.01	1.01	1.01				1.01	1.01	1.01			
bytes	4294	3683	3159				4305	3368	2671			
SNR	34.71	33.85	29.09				34.54	30.78	24.88			
α	--	1.04	1.04									
bytes	--	5480	3635									
SNR	--	32.96	29.03									

Comparison of experimental results for non-adaptive schemes for PLATH

Table 4.2.1

CAFE

	row-wise scanning											
	coding using crit. 1'						coding using crit. 2'					
	IEAR(1)			IEAR-IC(1)			IEAR(1)			IEAR-IC(1)		
	$\epsilon'=5$	$\epsilon'=8$	$\epsilon'=15$	$\epsilon'=5$	$\epsilon'=8$	$\epsilon'=15$	$\epsilon'=3$	$\epsilon'=7$	$\epsilon'=15$	$\epsilon'=3$	$\epsilon'=7$	$\epsilon'=15$
α	--	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
bytes	--	6179	5075	6372	5817	5175	--	5303	3985	6442	5593	4980
SNR	--	28.61	26.07	28.77	23.61	26.42	--	26.53	22.10	25.65	21.97	18.46
α	0.99	0.99	0.99	0.98	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.98
bytes	6025	5295	4394	6071	5795	5385	5898	4706	3595	6255	5637	4990
SNR	29.11	28.63	26.13	23.82	28.52	22.71	28.80	26.68	22.29	26.84	24.54	20.08
α	1.00	1.00	1.00	0.99	0.99	0.99	1.00	1.00	1.00	0.99	0.99	0.99
bytes	5798	5147	4292	6295	6001	5488	5772	4600	3545	6372	5688	4996
SNR	29.04	28.69	26.45	29.09	28.55	22.69	28.90	27.01	22.61	28.06	25.80	20.67
α	1.01	1.01	1.01				1.01	1.01	1.01			
bytes	5824	5215	4385				5837	4686	3614			
SNR	28.95	28.58	22.90				28.86	23.24	21.03			
α	--	1.04	1.04									
bytes	--	6145	4798									
SNR	--	23.50	22.74									

Comparison of experimental results for non-adaptive schemes for CAFE

Table 4.2.2

of α-values in this nbd., the value of α which gives the best compresion performance for a particular configuration of an algorithm, is first determined. A comparison of the compaction capabilities between the various possible configurations, for these values of α which yield the best compactions, is next made and the variant which yields the best results is identified. The procedure is repeated for other images, and if the same version of the algorithm yields the most desirable performance for different images, then this is the uniformly best scheme.

The performance of the different coding algorithms, for some representative values of α is shown in Tables 4.2.1-4.2.2, for different choices of ϵ', for the images CAFE and PLATH, and for the row-wise scanning case. Here the prediction error is quantized to 7 levels. Note that here actually eight levels are available for the quantization of the prediction error. The case of 7 quantization levels is being considered to provide a comparison with the adaptive methods and the results for a 8-level quanitzer are taken up later.

The tables 4.2.1 and 4.2.2 allow us to make the following statements about the relative performance of the coding schemes:

When the IEAR(1) process based coding scheme is used, for a given value of ϵ', the best cumulative compression and SNR peformance is obtained by using a value of α equal to 1.00, though in some sporadic cases the values 1.01 and 1.02 yield marginally better performance. The advantage of an α-value of 1.00 is that the multiplications in the transmitter and the receiver can be done away with, and time for computation need no longer be a bottle neck in increasing the transmission rates. For the IEAR-IC(1) process based coding scheme, the value of α, using which the best performance, for a given ϵ', is achieved, keeps varying from

image to image and also on the specific configuration of the algorithm used Typical α-values, for which good performance is attained, in case of different images, and different parameter values, are 0.96, 0.98 and 0.99. However, the IEAR(1) process based coding scheme with α=1.00 and which uses crit. 1' to indicate the ZE conditions performs considerably better than the three other schemes possible, viz. the IEAR(1) scheme with, α=1.00 and crit. 2' to indicate the ZE conditions, and the IEAR-IC(1) scheme, with a suitable α and using either crit. 1' or crit. 2' to signal the ZE conditions. Exactly the same conclusion is arrived at by using 6 bits/pel images and by scanning the image block-wise. A comparison between the Tables 4.1.1 and 4.2.1 and the tables 4.1.3 and 4.2.2, respectively, demonstrates the superiority of the non-adaptive IEAR(1) based coding scheme, with α=1.00, over any of the adaptive schemes. The chief reasons behind this fact are:

1. The adaptive schemes have to endure the brunt of the overheads that are required to signal the switch between the IEAR/IEAR-IC coding strategies, but convey no information to the receiver about the image itself.

2. The process of identifying the image regions as the sample functions of the IEAR/IEAR-IC processes, do so after ensuring that a saving over DPCM would be achieved on coding these regions using the IEAR/IEAR-IC based methods, but such an economy cannot be ensured in the presence of quantization errors. That is, the latter coding methods may actually turn out to be more wasteful than DPCM in the presence of quantization errors.

3. An 8 level quantizer is available in the case of the non-adaptive methods, as opposed to a 7 level one for adaptive schemes, and this allows the non-adaptive schemes to attain higher SNR's.

This means that the scheme which performs best, among the adaptive and the non-adaptive methods, is the IEAR(1) process based coding scheme with α=1.00 and

Row-wise scanning

	CAFE		PLATH		PLATHHQ			LISA		LINCOLN	
	bytes	SNR	bytes	SNR	bytes	SNR		bytes	SNR	bytes	SNR
$\epsilon'=5$	5750	29.41	4173	34.55	4733	27.80	$\epsilon'=2$	1369	18.38	1143	25.17
$\epsilon'=8$	5160	28.83	3526	33.33	4159	27.37	$\epsilon'=4$	1093	17.84	0961	22.77
$\epsilon'=15$	4303	26.51	2929	28.22	3453	25.06	$\epsilon'=7$	0841	16.41	0799	19.72

Block-wise scanning

	CAFE		PLATH		PLATHHQ			LISA		LINCOLN	
	bytes	SNR	bytes	SNR	bytes	SNR		bytes	SNR	bytes	SNR
$\epsilon'=5$	5883	25.86	4181	32.21	4780	25.01	$\epsilon'=2$	1355	21.59	1147	23.78
$\epsilon'=8$	5312	25.77	3662	31.06	4293	24.70	$\epsilon'=4$	1087	20.49	1147	23.78
$\epsilon'=15$	4417	24.58	2860	28.23	3447	23.64	$\epsilon'=7$	0818	18.16	0813	19.65

Performance of the IEAR(1) based coding method,

α=1.00 and using crit. 1′ to signal the ZE conditions

Table 4.2.3

Row-wise scanning

	CAFE		PLATH		PLATHHQ			LISA		LINCOLN	
	a	b	a	b	a	b		a	b	a	b
$\epsilon'=5$	6.4	0.16	32.1	1.20	23.0	0.35	$\epsilon'=2$	10.9	0.14	25.6	-0.34
$\epsilon'=8$	16.0	0.74	42.6	2.42	32.3	0.78	$\epsilon'=4$	28.8	0.25	37.4	1.06
$\epsilon'=15$	30.0	3.06	52.3	7.53	45.5	3.09	$\epsilon'=7$	45.2	0.75	48.0	4.48

Block-wise scanning

	CAFE		PLATH		PLATHHQ			LISA		LINCOLN	
	a	b	a	b	a	b		a	b	a	b
$\epsilon'=5$	4.2	0.08	31.9	0.4	22.2	0.07	$\epsilon'=2$	11.8	-0.27	25.3	-1.12
$\epsilon'=8$	13.5	0.17	49.0	1.55	30.1	0.38	$\epsilon'=4$	29.2	0.83	38.0	-0.47
$\epsilon'=15$	28.1	1.36	53.5	4.38	43.9	1.44	$\epsilon'=7$	46.7	3.16	47.1	1.94

Notation

a:Percentage improvement in compression over DPCM

b:Decrease in SNR over DPCM in dB

Performance of the non-adaptive IEAR(1) based coding method,

α=1.00 and using crit. 1' to signal the ZE conditions,relative to DPCM

Table 4.2.4

the 8 bits/pel images the values of ε' may be taken to be 5 and 8, respectively without undue loss in SNR. However, image areas with small changes in pel intensities are now replaced by constant intensity regions and this degradation is noticeable particularly in low intensity regions in the form of either streaks or blotches of constant gray levels.

4.3 Conclusions

This work has explored the possibilities of achieving high compressions, in the case of coding of images, by exploiting the structure of some models for non-gaussian time series. It was observed in sec. 2.11, that of an immensity of such models which have been taken up in literature, only the first order model, the IEAR(1) process, can be used to attain these ends. A new process, the IEAR-IC(1) process, was constructed and its coding performance compared vis-a-vis the IEAR(1) process.

Two schemes of image coding using schemes based on these methods were taken up: the adaptive and the non-adaptive one. For the scheme which changes the prediction coeffecients adaptively, it was found that the method using crit. 1 for identifying the sample functions IEAR(1)/IEAR-IC(1) processes and crit. 1' for signalling the occurance of the ZE conditions, almost always performs as good as, if not better than, any other adaptive scheme based on these processes. However, the arguments in section 4.2 reveal that the scheme which performs best, among all possible variants of the the adaptive and the non-adaptive methods, based on the IEAR(1) and the IEAR-IC(1) processes, is the IEAR(1) process based coding scheme with α=1.00 and which makes use of crit. 1' to signal the ZE conditions. The implementation complexity and the time for computation taken by this scheme is insignificant when compared with the complexity and the processing time for the adaptive schemes. For images consisting predominantly of large solid areas, for example, head and shoulder

the 8 bits/pel images the values of ϵ' may be taken to be 5 and 8, respecti\ without undue loss in SNR. However, image areas with small changes in pel intensit are now replaced by constant intensity regions and this degradation is noticea particularly in low intensity regions in the form of either streaks or blotches constant gray levels.

4.3 Conclusions

This work has explored the possibilities of achieving high compressions, the case of coding of images, by exploiting the structure of some models for n gaussian time series. It was observed in sec. 2.11, that of an immensity of su models which have been taken up in literature, only the first order model, the IEAR process, can be used to attain these ends. A new process, the IEAR-IC(1) process, w constructed and its coding performance compared vis-a-vis the IEAR(1) process.

Two schemes of image coding using schemes based on these methods were tak up: the adaptive and the non-adaptive one. For the scheme which changes th prediction coeffecients adaptively, it was found that the method using crit. 1 fc identifying the sample functions IEAR(1)/IEAR-IC(1) processes and crit. 1' fc signalling the occurance of the ZE conditions, almost always performs as good as, i not better than, any other adaptive scheme based on these processes. However, th arguments in section 4.2 reveal that the scheme which performs best, among al possible variants of the the adaptive and the non-adaptive methods, based on th IEAR(1) and the IEAR-IC(1) processes, is the IEAR(1) process based coding scheme wit α=1.00 and which makes use of crit. 1' to signal the ZE conditions. The implementatio complexity and the time for computation taken by this scheme is insignificant whe compared with the complexity and the processing time for the adaptive schemes. Fo images consisting predominantly of large solid areas, for example, head and shoulde

images with constant background, this method may give rise to compression ratios of almost 2 relative to DPCM. For images with a large amount of detail in them, small improvements over DPCM are achieved.

In this study, comparison of the IEAR(1)/IEAR-IC(1) process based schemes has been made with 3 bit DPCM. For higher bit rate DPCM schemes these methods will achieve better percentge compression over DPCM, because the occurance of a ZE condition can still be indicated using one bit, and the fraction of saving made relative to DPCM signalling of this condition would improve. However, it must be remembered that the scheme is a variable length coding scheme, with all the concomitant problems, and for noisy channels it may become imperative to use a error control scheme in conjunction with this scheme, and this may negate the compressions achieved somewhat. It is suitable for image storage purposes where the problems associated with the presence of noise are virtually non-existent.

The IEAR(1) process based coding method, with $\alpha=1.00$ is, under certain circumstances, identical to a special case of a variable length coding scheme, which can be used to code the quantized prediction error values obtained in the case of DPCM, with the predictor coeffecient $\alpha'=1.00$. To see this, consider the 7-level quantizer for 8 bits/pel images, given in sec. 3.6. This quantizer quantizes the interval [-5,5] to zero. If, in the mentioned IEAR(1) process basesd scheme, ϵ' is chosen to be 5, then this scheme is identical to a coding scheme which assigns variable length code words, to the quantized prediction error obtained in the case of DPCM, in the following way: it assigns a one bit word to the zero quantized prediction error and assigns a four bit word (one bit indicating that this level is not equal to zero and three bits giving the reconstruction value) to the other quantization values. The above argument holds for any quantizer having an odd number of reconstruction levels, i.e. which quantizes a certain interval to the zero

value. For a 8-level quantizer, choosing ϵ' greater than or equal to 8, implicitly converts the 8 level quantizer to a 7 level quantizer and is again equivalent to the above variable length coding scheme, a choice of ϵ' less than 8 actually converts the 8 level quantizer to a 9 level quantizer and the above argument is again valid. Thus we see that the scheme with the best compression performance turns out to be simply a variable length coding scheme, which can be used to code the quantized prediction error values, obtained in the case of a DPCM scheme which makes use of the same quantizer as the IEAR(1) process based scheme.

This study, thus disputes the claim that superior coding of images can be achieved by the schemes based on the IEAR(1)/IEAR-IC(1) processes, as in [1], and establishes that the scheme which performs best, within this family of schemes, is in no way different from a simple variable length coding scheme which can be used in conjunction with DPCM.

4.4 A note on the visual degradation of images encoded using the IEAR(1)/IEAR-IC(1) process based schemes

The images encoded using the procedure based on the IEAR(1)/IEAR-IC(1) process have edges which are not as sharp as those in the original image. This jaggedness or blurring of edges causes an overall decrease in sharpness in the appearance of the image. This phenomenon is similar to that occuring due to the quantization errors in the case of DPCM but is aggravated in this case, particularly if ϵ' is chosen large.

Quantizing the range $[-\epsilon',\epsilon']$ as zero has the effect of causing areas of equal intensity, with little variation, and gives rise to horizontal streaks if the image is scanned row-wise and blotches of constant intensity if it is scanned block-wise. Such

4.5 Suggestions for further work

The results in this thesis would preclude much further work in applying the IEAR process for image coding applications. However, a direct extension of this work would be to compare the above results, obtained in the IEAR(1)/IEAR-IC(1) process based coding of images, with the Huffman coding of the quantized prediction error values. Huffman coding could be achieved after obtaining the estimates of the probability with which a particular reconstruction level is reached, duing the quantization process. Studies could also be undertaken on the rate distortion theory of these sources.

The above pessimism, however, should not affect the general interest in the IEAR process itself. For one, these processes were not constructed with coding applications in mind. The process and its variants are handy in modelling data which are known to be patently non-gaussian, like intervals and counts. Also, the process provides a way of generating non-gausian time series, without resorting to non-linear sytems. From, a theoretical and a practical viewpoint, this research would take the directions suggested by [15]-[20].

References

1. J. D. Gibson, *"Data Compression of a First Order Intermittently Excited AR process"*.

2. A. K. Jain, *"Image Data Compression: A Review"*, Proc. IEEE, Vol. 69, March 1981.

3. A. N. Netravali and J. O. Limb, *"Picture coding: A Review"*, Proc. IEEE, Vol. 68, March 1980.

4. H. G. Musmann, P. Pirsch and H. J. Gallert, *"Advances in Picture Coding"*, Proc. IEEE, Vol. 73, April 1985.

5. W. K. Pratt, *"Digital Image Processing"*, 2nd Edition, Wiley Interscience.

6. A. K. Jain, P. M. Farelle and V. R. Algagi, *"Image Data Compression"*, in Digital Image Processing Techniques, Academic Press.

7. W. F. Schreiber, *" The measurements of 3rd order probabililty distributions of TV signals"*, IRE Trans. Inf. Th., IT-2, Sept. 1956.

8. M. Kunt, A. Ikonomopoulos and Kocher, *"Second Generation Image Coding Techniques"*, Proc. IEEE, Vol. 73, April 1985.

9. Robert M. Gray, *"Vector Quantization"*, IEEE ASSP Magazine, April 1984.

10. D. J. Granrath, *"The role of Human Visual Models in Image Processing"*, Proc. IEEE, Vol. 69, May 1981.

11. C. L. Mallows, *"Linear Processes are nearly Gaussian"*, J. App. Prob., Vol. 4, 1967.

12. T. Berger, *"Rate Distortion Theory"*, Prentice Hall, 1971.

13. A. Papoulis, *"Probability, Random Variables and Stochastic Processes"*, McGraw Hill, 1984.

References

1. J. D. Gibson, *"Data Compression of a First Order Intermittently Excited process"*.

2. A. K. Jain, *"Image Data Compression: A Review"*, Proc. IEEE, Vol. 69, March 1981.

3. A. N. Netravali and J. O. Limb, *"Picture coding: A Review"*, Proc. IEEE, Vol. March 1980.

4. H. G. Musmann, P. Pirsch and H. J. Gallert, *"Advances in Picture Coding"*, Pro IEEE, Vol. 73, April 1985.

5. W. K. Pratt, *"Digital Image Processing"*, 2nd Edition, Wiley Interscience.

6. A. K. Jain, P. M. Farelle and V. R. Algagi, *"Image Data Compression"*, in Digit Image Processing Techniques, Academic Press.

7. W. F. Schreiber, *" The measurements of 3rd order probabililty distributions TV signals"*, IRE Trans. Inf. Th., IT-2, Sept. 1956.

8. M. Kunt, A. Ikonomopoulos and Kocher, *"Second Generation Image Codi Techniques"*, Proc. IEEE, Vol. 73, April 1985.

9. Robert M. Gray, *"Vector Quantization"*, IEEE ASSP Magazine, April 1984.

10. D. J. Granrath, *"The role of Human Visual Models in Image Processing"*, Pro IEEE, Vol. 69, May 1981.

11. C. L. Mallows, *"Linear Processes are nearly Gaussian"*, J. App. Prob., Vol. 1967.

12. T. Berger, *"Rate Distortion Theory"*, Prentice Hall, 1971.

13. A. Papoulis, *"Probability, Random Variables and Stochastic Processes"*, McGra Hill, 1984.

14. D. P. Gaver and P. A. W. Lewis, *"First Order AR Gamma Sequences and Point Processes"*, Adv. App. Prob., Vol. 12, 1980.

15. A. J. Lawrence and P. A. W. Lewis, *"The Exponential ARMA, EARMA(p,q) process"*, J. Royal Stat. Soc., Vol. 42, 1982.

16. A. J. Lawrence and P. A. W. Lewis. *"A New AR time series model in Exponential Variables"*, Adv. App. Prob. Vol. 13, 1981.

17. A. J. Lawrence and P. A. W. Lewis,*"A Mixed Exponential time series model"*, Management Science, Vol. 28, 1982.

18. Don. H. Johnson and P. Srinivas Rao, *"Generation of Linear First Order time series"*.

19. Don. H. Johnson and P. Srinivas Rao, *"Proprties and Generation of Non-gaussian time series"*.

20. Don. H. Johnson and P. Srinivas Rao, *"A First Order AR model for Non-gaussain time series"*.

21. J. Makhoul, *"Linear Prediction: A Tutorial Review"*, Proc. IEEE, Vol. 63, April 1975.

22. A. K. Jain, *"Advances in Mathematical Models in Image Processing"*, Proc. IEEE, Vol. 69, May, 1981.

Reconstructed images

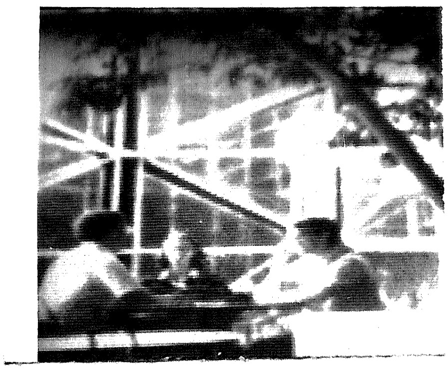
The images included in this section have been coded and reconstructed using the specific version of the algorithm indicated alongside each reproduced image. The number of bytes required to code the image and the SNR's obtined are also indicated. While considering the degradations caused in the images, the observations made in sec. 4.4 must be borne in mind.

PLATH

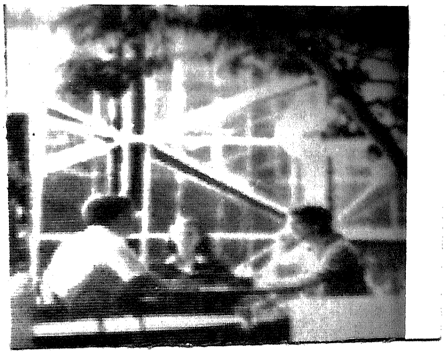
1. Original

2. Coded using DPCM
Row-wise scanning
SNR=35.75 dB, bytes=6144

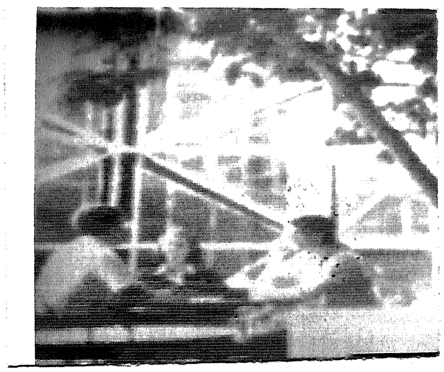
CAFE



1. Original

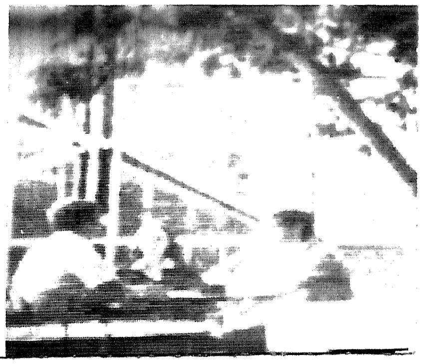


2. Coded using DPCM
Row-wise scanning
SNR=29.57 dB, bytes=6144

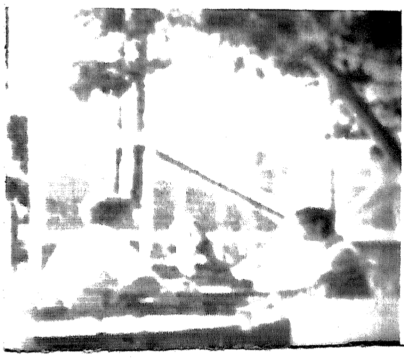


3. Coded using non-adaptive IEAR(1) based scheme, α=1.00, using crit. 1', ϵ'=15
Row-wise scanning
SNR=26.51dB, bytes=4303

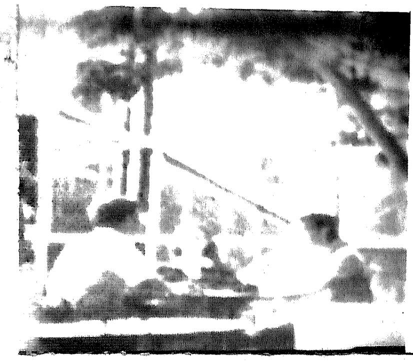
4. Coded using non-adaptive IEAR(1) based scheme, α=1.00, using crit. 1', ϵ'=15
Block-wise scanning
SNR=24.58 dB, bytes=4417



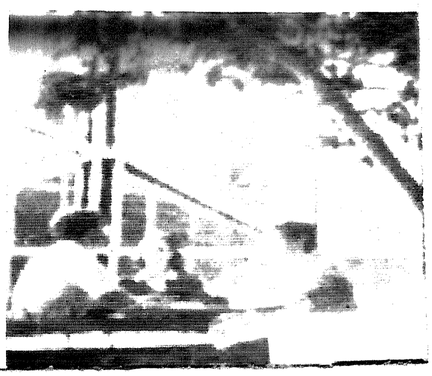
5. Coded using the adaptive IEAR(1) based scheme, using crit. 2, crit. 2', $t \geq n(l)$
$\epsilon=4$, $\epsilon'=15$
Row-wise scanning
SNR=25.59 dB, bytes=5186



6. Coded using the adaptive IEAR(1) based scheme, using crit. 2, crit. 1', $t \geq n(l)$
$\epsilon=4$, $\epsilon'=15$
Row-wise scanning
SNR=27.57 dB, bytes=5406



7. Coded using the adaptive IEAR-IC(1) based scheme, using crit. 1, crit. 1', $t \geq n(l)$
$\epsilon=5$, $\epsilon'=8$
Row-wise scanning
SNR=28.88 dB, bytes=5634



8. Coded using the adaptive IEAR-IC(1) based scheme, using crit. 1, crit. 1', $t \geq n(l)$
$\epsilon=15$, $\epsilon'=15$
Row-wise scanning
SNR=26.89 dB, bytes=5188